Tasks to promote argumentation in math class based on Dynamic Geometry Software[1]

Tareas para promover argumentación en clase de matemáticas basadas en Software de Geometría Dinámica

Tarefas para promover a argumentação na classe de matemática baseada no Software de Geometria Dinâmica

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Abstract

Purpose: The tasks proposed to students have an impact on the cognitive activity they develop and on the construction of concepts and joint meanings. In this article we propose two tasks to promote interactivity in class mediated by argumentation processes with the use of GeoGebra.

Description: Dynamic geometry environments allow students to experiment with different types of semiotic representation to mediate the cognitive activity they develop. Although some studies suggest that these environments decrease the cognitive load, it is recognized in the literature the impact that this type of systems generate in school learning, such as their use promotes the development of strategies to extend the problem or explore cases that allow students to generalize on a hypothesis or conjecture. In this sense, the importance of entrainment and of the concepts, definitions and fundamental topics of Euclidean geometry that are necessary for the constructions of the models that are tested is highlighted.

Point of view: The teacher's mediation and the presentation of significantly rich and challenging tasks allow generating moments of interactivity where the teacher's actions are articulated with those of the students around the task.

Conclusions: it is important for teachers to design activities that promote argumentation spaces in class as an opportunity to learn mathematics, since it allows confronting different points of view and mediating in the construction of meanings.

Keywords: Argumentation, Modeling, Social Interaction, Interactivity, Task Design

Resumen

Propósito: Las tareas que se proponen a los estudiantes repercuten en la actividad cognitiva que desarrollan y en la construcción de conceptos y significados conjuntos. En este artículo proponemos dos tareas para promover la interactividad en clase mediadas por procesos de argumentación con el uso de GeoGebra.

Descripción: los entornos de geometría dinámica permiten que los estudiantes experimenten con diversos tipos de representación semiótica para mediar la actividad cognitiva que desarrollan. Si bien, algunos estudios sugieren que estos entornos disminuyen la carga cognitiva, se reconoce en la literatura el impacto que este tipo de sistemas generan en el aprendizaje escolar, su uso promueve el desarrollo de estrategias para ampliar el problema o explorar casos particulares que les permita a los estudiantes generalizar sobre una hipótesis o conjetura. En ese sentido, se resalta la importancia del arrastre y de los conceptos, definiciones y tópicos fundamentales de la geometría euclidiana que son necesarios para las construcciones de los modelos que se someten a prueba.

Punto de vista: la mediación del profesor y la presentación de tareas significativamente ricas y desafiantes permiten generar momentos de interactividad donde se articulan las acciones del profesor con las de los estudiantes en torno a la tarea.

Conclusiones: es importante que los profesores diseñen actividades que promuevan espacios de argumentación en clase como una oportunidad para aprender matemáticas dado que permite confrontar diversos puntos de vista y mediar en la construcción de significados.

Palabras clave: Argumentación, Modelación, Interacción Social, Interactividad, Diseño de tareas

Resumo

Objetivo: As tarefas propostas aos alunos têm um impacto na atividade cognitiva que desenvolvem e na construção de conceitos e significados comuns, neste artigo propomos duas tarefas para promover a interatividade na aula mediada por processos de argumentação com a utilização do GeoGebra.

Descrição: os ambientes de geometria dinâmica permitem aos estudantes experimentar diferentes tipos de representação semiótica para mediar a atividade cognitiva que desenvolvem. Embora alguns estudos sugiram que estes ambientes diminuem a carga cognitiva, é reconhecido na literatura o impacto que este tipo de sistemas gera na aprendizagem escolar, a sua utilização promove o desenvolvimento de estratégias para expandir o problema ou explorar casos particulares que permitem aos estudantes generalizar sobre uma hipótese ou conjectura. Nesse sentido, destaca a importância do arrasto e dos conceitos, definições e tópicos fundamentais da geometria euclidiana que são necessários para as construções dos modelos que são testados.

Ponto de vista: a mediação do professor e a apresentação de tarefas significativamente ricas e desafiantes permitem gerar momentos de interatividade onde as ações do professor são articuladas com as dos alunos em torno da tarefa.

Conclusões: é importante que os professores concebam atividades que promovam espaços de argumentação na aula como uma oportunidade para aprender matemática, dado que lhes permite confrontar diferentes pontos de vista e mediar na construção de significados.

Palavras-Chave: Argumentação, Modelação, Interação Social, Interatividade, Desenho de tarefas

Approach to the problem

One way to address the mathematical reasoning of students is by analyzing the arguments they provide. Discursive activity in the classroom becomes relevant to the extent that it is linked to the student in the construction of knowledge (Ríos-Cuesta, 2020). Several researchers have been responsible for documenting some elements and conditions of the argumentation in class along with the teacher's role in the validation of student productions. However, we analyze the case of a public school in the department of Chocó (in Colombia), and we find that it is difficult for students to present a valid argument as the result of a task and that they use mechanically algorithms that are extracted from textbooks (Ríos-Cuesta, 2021a). It is evident that there is no understanding of the steps they follow when solving a problem or how the algorithms they use are constructed.

The previous scenario led us to analyze the results of the standardized tests carried out by the Ministerio de Educación Nacional [MEN], through the Instituto Colombiano para Evaluación de la Educación [ICFES], using the *Saber Test* as a mechanism to collect information on the status of students in the third, fifth, and ninth grade. This test verifies the proficiency levels presented by the students. In the case of mathematics, the competences evaluated are, Problem solving, Reasoning and Communication. The results of the *Saber Test* highlight a difficulty in the training of students. In table 1, we present the results of ninth grade which were published by MEN (2018) in the four-year report.

Table 1. Results Saber test of the department of Chocó

Competence	Percentage of incorrect answers			
	2014	2015	2016	2017
Resolution	64.9	68.0	65.2	70.7
Reasoning	62.2	66.1	62.0	68.2
Communication	59.4	68.3	62.4	68.4

Source: own elaboration

In the previous results we noticed that the students of the department or state of Chocó are not reaching or developing the necessary skills in the tests carried out by the ICFES, nor is there a consolidation of mathematical knowledge that allows them to face or succeed in said test.

On the other hand, the fourth level in the Programme for International Student Assessment (PISA test) corresponds to the ability to communicate explanations and arguments based on their interpretations, in the fifth level are students who can develop models and work with them (Rico, 2007). The PISA test conceives argumentation as the ability to differentiate various types of reasoning and create and express mathematical arguments (Organization for Economic Cooperation and Development [OECD], 2017). Colombian students are not doing very well in this test. It is reported that in 2015, only 11.4% reached levels 3 and 4, and 0.3% levels 5 and 6 (Agencia de Calidad de la Educación, 2015). The average of Colombian students was lower than that of the OECD in 2018, 35% of students were placed at level 2 or higher, of which, about 1% were between levels 5 and 6 (OECD, 2019). Now, level 2 is considered as the minimum level that students must reach, below that level 66% of Colombian students who took the test in 2018 are located (ICFES, 2020). Figure 1 shows the evolution of the average in Colombia.

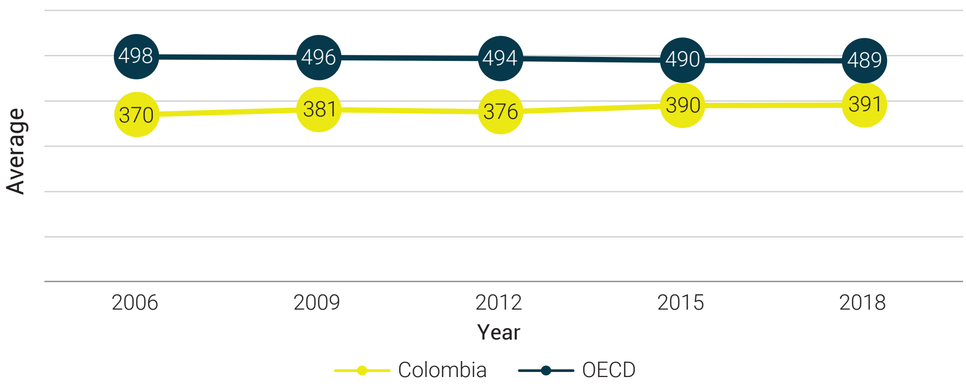

Figure 1. Evolution of the average in Colombia in the Mathematics Test
Source: own elaboration based on PISA data.

We consider that attention should be paid to argumentative practices as an opportunity to develop mathematical knowledge in high school students, classroom practices and the results of standardized tests show that students find it difficult to argue the result of the activity they develop in class, this makes it difficult to include more complex mathematical tasks and consequently the activities developed in class are limited to the practice and execution of algorithms prescribed by the teacher (Goizueta, 2015). On the other hand, Cervantes-Barraza et al. (2019) report that students from early ages have difficulties in arguing using mathematical elements that justify their reasoning, some of these results fall on the classroom management performed by the teacher to sustain and encourage discussions and develop tasks that promote argumentation.

In accordance with the above, it is important to highlight the role that tasks play in the construction of joint understandings of the mathematical objects that are developed in the classroom. No less important is the development of argumentation processes that allow students to propose alternative solutions to tasks that admit different heuristics. This type of tasks provokes debate and confrontation of ideas, which can be used by the teacher to mediate in the cognitive processes of the students and help them build ways of thinking, according to the mathematical activity and concepts at stake.

The aim of this article is to propose two tasks that have the potential to promote interactivity in the math classroom and to generate moments of discussion around the solution of the task. These tasks have been designed to be implemented in GeoGebra since they allow students to move between different registers of semiotic representation. In addition, elements are provided for the design of other tasks that encourage discussion and generate argumentation in class. Duval (2017) defines semiotic representation registers as the different writing systems, whether numerical, symbolic, or graphical that allow the communication and objectification of the mathematical object.

The importance of semiotic representation registers in the learning of mathematics

Valbuena-Duarte et al. (2021) conducted a study in which they identified that students solve tasks using only one semiotic representation system and present difficulties in moving from one representation register to another; these conversions between systems are the ones that generate the most problems for students. However, they apply a didactic strategy that shows evidence that the use of GeoGebra software allows students to move between representation registers using the graphic register as a semiotic mediator.

On the other hand, the articulation of concepts is another problem that students manifest when developing tasks with high cognitive demands. Cervantes-Barraza et al. (2020) found that students resort to empirical strategies that sometimes lack conceptual and axiomatic support. However, Villagómez, Ruiz and Acosta (2020) conducted a study where they propose tasks that allow the transition from arithmetic to algebra using semiotic representation registers finding that students use the verbal register mediator to understand the problem, and this guides the actions of the graphic register. A characteristic of the proposed tasks is the use of verbal problems.

Moving between different registers of semiotic representation using software helps the formation of concepts in students and motivates the learning of mathematics for which the teacher must propose tasks that move away from textbooks (Báez-Ureña, Pérez-González & Blanco-Sánchez, 2018).

Argument as a social practice

We have shown a panorama where argumentation is conceived as the discussion activity that develops in the classroom through the interactions of students to validate and build mathematical school knowledge (Ríos-Cuesta, 2021b). In that sense, the teacher is placed as the person in charge of the management of student participation shifts or opportunities. Given its social, verbal, and rational nature, it promotes the use of cognitive tools and resources (Hoyos, 2018; Ríos-Cuesta, 2020).

Ruiz (2012) argues that the argumentation in the math classroom fulfills three fundamental purposes which relate to intentionality. The first of these refers to the negotiation of meaning and construction of communities of practice. The second has to do with the discursive interaction which affects the subject's cognition as he develops competencies that allow him to see society differently. The third refers to the role played by the participants since everyone's contribution in the construction of knowledge is valued. On the other hand, Ruiz (2012) in the same thesis, recognizes the role of argumentation as an instrument for the construction of knowledge and that, through interaction with the other, offers the possibility to listen to their ideas, analyze them and discuss their content.

The classroom is a space that goes beyond the transmission of knowledge by the teacher to the students, it is so complex that it must focus on the critical reflection of the situations that arise by encouraging reasoning and argumentation in the students. This vision allows them to face thought processes in which social interaction makes great sense, to the point that he becomes the protagonist of mathematical activity. Students expose their ideas, the teacher somehow validates them, and mathematical knowledge is built. On this aspect, Rasse and Solar (2019) comment that a mathematics learning based on argumentative practices allows students to verbalize their reasoning and develop argumentation skills, in addition, the teacher can identify conceptual errors in students when solving tasks, applying theorems or formulas.

Authors such as Jiménez-Aleixandre (2010) and Ruiz (2012) argue that inside the classroom, spaces should be created for students to make their reasoning and thoughts public, where debate, presentation, and justification of evidence are favored. This implies a change in the role of the student and the teacher, their practice, and the contents to be taught. On the student's role change, he goes from being a passive subject to having a reflexive vision about his learning, the way he reasons mathematically and how he builds knowledge. Consequently, it consolidates its reasoning through verbalization and interaction with others (McCrone, 2005; Chico, 2014). In this sense, Valbuena-Duarte et al. (2020) conclude that, within the math classroom, a reflective environment should be created where students can construct formal knowledge based on tasks that encourage participation.

Kukliansky (2019) argues that teaching based on the development of arguments in the math class helps students understand the relevant concepts by allowing the cognitive part and social interaction to develop. Thus, Krummheuer (2015) conceives argumentation as a condition for learning mathematics that depends on the participation of students and on the commitment to building knowledge through explanatory and justifying practices.

Approach to recent studies

Among the recent works developed in the line of reasoning, argumentation, and proof in mathematical education, we highlight Erkek and Işıksal-Bostan (2019), who studied the interactions in the math classroom, engaging students with justification in the class. The use of GeoGebra allowed students to analyze in more detail the movement of the segments, thereby promoting other forms of argumentation.

On the other hand, Solar (2018) develops a study on the implications of collective argumentation in the classroom. It was based on the Toulmin model and studied the teacher's role in that process. Among the implications of the argument is the identification of various ways of thinking, the dialogic interaction that takes place between the teacher and the students and the tools that allow addressing contingencies in the classroom, these three provide opportunities to promote social justice. Among its findings, it is highlighted that it is possible to deepen the students' learning using the error as a starting point, and much importance is given to the role played by the refutator or contradictor in the Toulmin model. Another recent study developed by Cervantes-Barraza et al. (2017) focused on analyzing the role of refutation and used Toulmin's model to reconstruct classroom arguments. These authors found two functions of refutation in the construction of mathematical knowledge, the first refers to the persuasive power when a formal guarantee is offered, and the second, to highlight the insufficiency of the guarantees used when establishing assertions by another arguer.

Other researchers such as Cervantes-Barraza and Cabañas-Sánchez (2018) offer a study on geometric argumentation using the Toulmin model. Their research methodology was based on the design research paradigm with a qualitative approach through a teaching experiment. It stands out among its findings that students use diagrammatic arguments which are associated with the visualization of the object of study, but they lack mathematical support, this is proof that students move away from the formalism of mathematics. However, the authors suggest the use of tasks in which exploration is encouraged through arguments based on guarantees.

Berciano et al. (2017) presented a study on reasoning and argumentation strategies in solving geometric problems in early childhood education with children between 3 and 5 years old. They used van Hiele's model and articulated it with Duval's apprehension levels offering a categorization of student arguments. These authors identify that the different phases followed by the teacher and the questions that generate reflection require the student a greater level of understanding of the object of study, encouraging in him a more complex argumentation that allows him to reach level 2 of van Hiele and an apprehension Operational that is not typical of the child's educational stage. We highlight the role played by the questions that the teacher asks to encourage the explanation of reasons by students. The goal of these questions is for students to improve their argumentation.

Estrella et al. (2017) conducted a study on statistical argumentation in students between first through fourth grades using the Toulmin model. They focus on the representations constructed by the students seeking to categorize them according to the logical, numerical, and geometric component of them. The methodology of the study was based on the proposition of open tasks in which students could do a heuristic exploration to organize data, subsequently they were given a semi-structured clinical interview making changes in the constructions offered by the students to trigger subtasks that were Associate some of the analysis categories. Among their findings, it

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is worth highlighting those external representations are “objects to think about” that serve as evidence to study argumentation fulfilling a function greater than communicative. Students use different types of representations to argue verbally, gesturally, and metaphorically by making associations with physical elements such as the ground and thus showing a spatial orientation.

Saorín et al. (2017) offer an analysis on the change of status of the propositions that make up the argumentation in solving geometry problems to prove. For the analysis they make use of the three types of apprehension proposed by Duval. Within the configural reasoning, three situations are revealed: truncation, conjecture without demonstration and the loop. In truncation the idea that solves the problem appears because of a deductive process; In the conjecture inferences are produced that help to solve the problem based on unproven hypotheses and the loop that is considered a stagnation when through the affirmations it is not possible to advance in the resolution of the problem. These researchers identify that the influence of prototypical configurations in the identification of some sub-configurations can lead to a reasoning that is not necessarily based on the data of the statement. On the other hand, when it is not possible to achieve a change of status in the statements of speech, a loop occurs. On the other hand, when the reasoning is based on an unproven conjecture, a change of status is generated in the statements that constitute a deductive process that leads to a solution to the problem. Finally, the argument serves as a bridge between the different apprehension cycles proposed by Duval in solving geometric test problems.

Methodological approach

The tasks presented have been designed with the objective of favoring argumentation as a way for students to acquire mathematical knowledge. This, in turn, provides elements for teachers to design other tasks considering the context and the students’ previous knowledge. In addition, it is proposed that class management should focus on the search for moments of interactivity in which the teacher’s actions are articulated with those of the students around the task and that the teacher’s returns of problems should focus on generating cognitive imbalances which should allow the construction of zones of proximal development for the teacher to mediate in them and favor learning.

Design of tasks with technology

When designing a task, the level of cognitive demand implicit in it should be considered to analyze the type of knowledge required by students. We start from the aspects defined by Doyle (1983), regarding the constituent elements of a task, among them, the goal, the resources, and the operations on the resources to achieve the goal.

Ramos-Rodríguez et al. (2015) synthesizes the work of Stein et al. (2009), on the cognitive demands in the tasks proposed to students and emphasizes that the teacher's management supports or inhibits student participation (see Table 2).

Table 2. Characterization of the cognitive demand of the tasks

Level of cognitive demand	Type	Description
Low cognitive demand	Memorization	Automatic classroom tasks, only memory is required, without performing procedures.
	Procedures without connections	Algorithms that require the use of a procedure, but do not require making connections between mathematical concepts.
High cognitive demand	Procedures with connections	They involve several underlying mathematical concepts, with multiple representations, that help develop the meaning of the task. They cannot be solved carelessly, and their answer is not explicitly deduced from the statement.
	Doing mathematics	Demands complex and non-algorithmic thinking, in which the nature of mathematical concepts must be explored and understood. It requires continuous analysis of the task, use of prior learning and self-regulation of cognitive processes.

Source: Ramos-Rodríguez et al. (2015).

However, in the same study, Ramos-Rodríguez et al. (2015) offer an extension of this characterization of the cognitive demands related to the tasks proposed in class and add a series of subcategories (see Table 3), these subcategories allow planning the teaching intervention and offer the necessary help to overcome the cognitive conflicts generated in the approach to the task. On the other hand, the subcategories help in the planning of other tasks with the same characteristics according to the learning purposes.

Table 3. Categories and subcategories of the cognitive demand of tasks

Categories	Subcategories	
Purpose	Addresses the problem Somewhat addresses the problem Does not address the problem	
Cognitive demand	Low	Memorized
		Procedure without connection
	High	Procedure with connection
		Doing mathematics
Resources	Operations (educational use)	Verifier of results. Reinforcer of mathematical ideas or concepts. Construction of mathematical ideas or concepts.
	Work format or grouping	Individual In groups Collective (whole class)
Mathematical content	Representations	Graphs Tables Diagrams Verbal
		Algebraic Numerical
	Role of the variable	As an unknown As a generalized number As a relationship between quantities

Source: Ramos-Rodríguez et al. (2015).

On the other hand, the use of technology, and in particular, the use of Dynamic Geometry Software (DGS) allows students to build and visualize concepts, recreate theorems using various semiotic representation systems and perform proofs arguing their conjectures, these favors learning to be more lasting and in turn, allows the relationships between the concepts that are put into play to be established (Zengin, 2018).

Modeling tasks proposed to favor argumentation

The tasks presented to the students require that the students use different representations to solve it, then we present some of the proposed tasks:

> Producing a cake costs $ 2, Mrs. Margarita sells the cakes for $ 4 each. She needs a person to help her sell them and pays her $ 20 per day. Normally

sells 100 cakes daily. A price increase of $ 0.5 makes you lose 5 customers, what would be the right price to maximize profits?

From the previous task students are expected to argue about the optimal price that must be set to optimize profit and production, for this, the sales function must be modeled as $v(x) = 100 - 5x$ and price as $p(x) = 4 + 0.5x$ then define the gain as $g(x) = 400 + 30x - 2.5x^2$ which results from the product of sales and price. The exploration promoted by GeoGebra software leads them to a solution like the one presented in the figure 2.

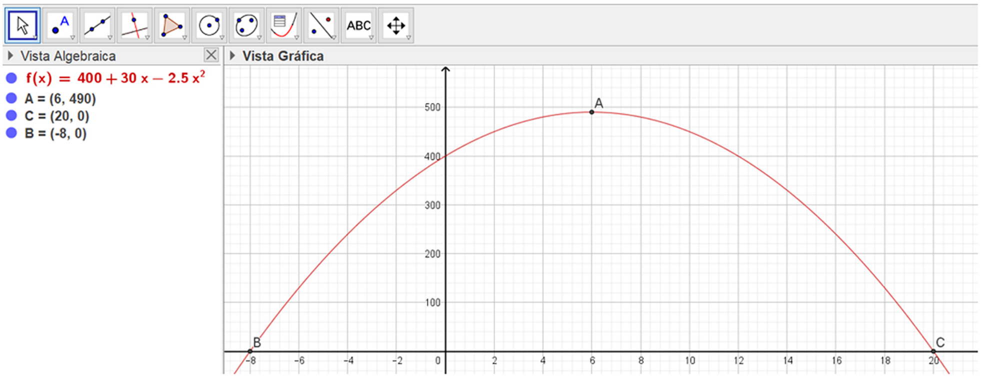

Figure 2. Solution offered in GeoGebra
Source: own elaboration

The student's actions should lead him to determine the coordinates of the maximum point and the gain that would be obtained in that case. The teacher should promote the discursive activity of the students so that they make public their reasoning and share their observations. In the case presented, the price that allows to have the maximum profit is $ 6 to obtain a profit of $ 490.

In the second task, it is proposed: Build a satellite dish using GeoGebra, explain how the signal is captured for the antenna to work. In this task, the student has the freedom to construct a parable with the parameters they wish to use, the discussion can be guided so that the students demonstrate Deositeo's theorem about the incidence of lightning (see figure 3). It is important the role of the teacher to guide students in conceptualization and testing. Next, we present the result of the task:

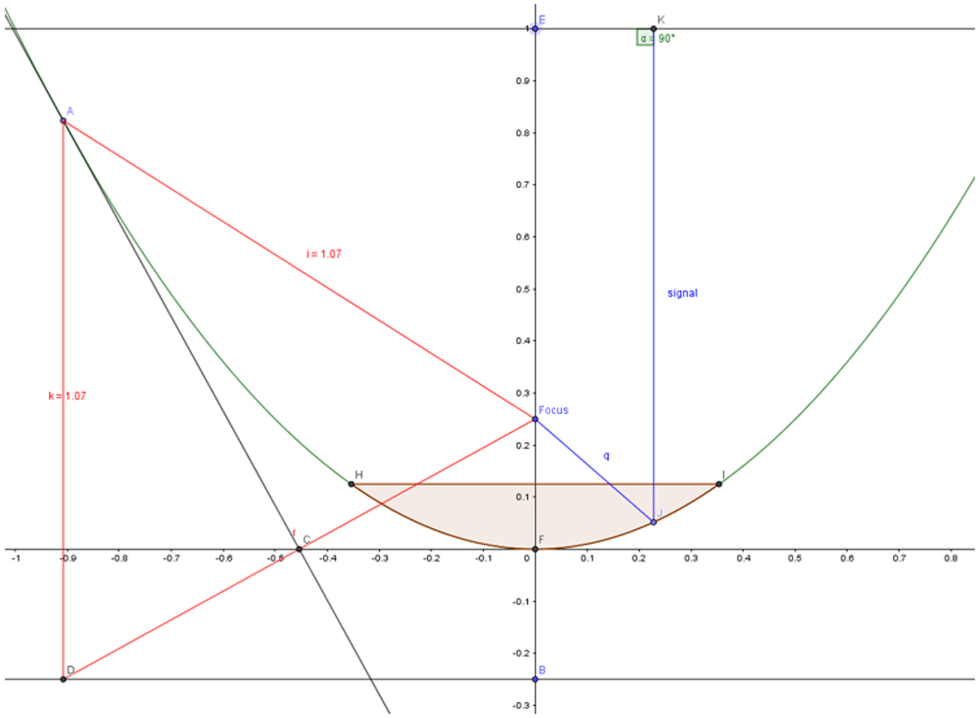

Figure 3. Parabolic antenna construction and the incidence of focus rays
Source: own elaboration

The success of the proposed tasks lies in the discussion of assignments in class to provoke cognitive conflicts, a term used by Piaget (1975) to refer to the imbalance between assimilation and accommodation of a conceptual scheme that is generated in students when facing a situation in which their previous knowledge is insufficient to achieve the necessary coordination to achieve the objective of the action, causing a cognitive imbalance which will seek to compensate with new abstractions and generalizations by modifying their cognitive structure, the above through a process of assimilation and accommodation in response to the balancing process. For this, the teacher's mediation is required to provide the necessary help to overcome the cognitive imbalances and to overcome the errors.

Conclusions

In this paper, we have tried to draw attention to the type of activities proposed to students to build mathematical knowledge. We propose as a central activity the generation of arguments where interactivity is promoted, a competence that high school students must develop is to model situations in different contexts.

The literature shows that, to generate argumentation in class, the teacher must make a good error management and use it as a learning tool. Plus, the questions asked must at least cause several aspects of mathematical practice to be considered, where the teacher must offer opportunities for participation to students. Another important factor is the type of tasks, which should favor the use of different procedures, that open responses are reached and that students can take different positions (Deulofeu, 2018).

Regarding the problems that allow different positions to be taken, modeling studies and verbal problems offer an opportunity for students to interact with mathematics in solving problems, which in turn, generates argumentative possibilities when validating their productions. We still have a long way to go in this regard, but the articulation of these two strong lines of research to improve the learning of high school students and their performance in standardized tests is what motivates this study.

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