Innovation in the Design of Four-Bar Mechanisms (FBM): integrated application of Hlawka-Grashoff inequalities together with PID type controllers with MatLab

Innovación en el diseño de mecanismos de cuatro barras (FBM): aplicación integrada de las desigualdades de Hlawka-Grashof junto con controladores tipo PID con MatLab

Inovação no projeto de mecanismo de quatro barras (FBM): aplicação integrada de desigualdades de Hlawka-Grashof com controladores PID usando MatLab

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Abstract

Introduction: This research carries out an applied and exploratory study of four-bar mechanisms (FBM), focusing on geometric constraints for an advanced analysis that facilitates the understanding of design, construction, and kinematics, aimed at practical applications in engineering and education.

Problem: Traditionally, FBM design has been based on the theory of mechanisms and machines, without incorporating advanced geometric mathematical constraints such as the Hlawka inequality. This omission limits the ability to comprehensively optimize these systems for specific applications.

Objective: Develop software in Matlab that integrates mathematical criteria based on the Hlawka and Grashof inequalities, also applying the Lagrangian for kinematic analysis and PI, PD, and PID controllers for system dynamics using FBM.

Methodology: An integrated approach that combines geometric, kinematic, and control analysis was used for FBM, developing an algorithm based on the Hlawka inequality. It was complemented by the creation of software in Matlab that adjusts drivers according to Grashof's law. Validation was performed using graphical comparisons and mean absolute error (MAE) analysis with a relevant case study.

Results: This project highlights the potential of an innovative mathematical approach in the design of four-bar mechanisms (FBM), significantly enriching their practical and training applications. Furthermore, it sets an important precedent for future research, proposing new avenues of study and exploration in advanced kinematics and mechatronic design, thus paving the way for innovative developments in fields related to engineering and technical education.

Conclusion: This project establishes a baseline for applications of FBM synthesis in control implementations and brings the user closer to the manipulation of parameters and variables through software that guarantees the understanding of the phenomenon for the design and construction of devices.

Originality: This study introduces a novel approach by integrating the Hlawka inequality into FBM design, establishing a pioneering framework for future research and technological development.

Limitations: Restrictions of the study arose from the paucity of specific comparative data available, underscoring the need for future research for further evaluation.

Keywords: Four-bar mechanisms, Hlawka inequality, Grashof's law, Matlab software, PID controllers, Mechatronic design.

Resumen

Introducción: esta investigación realiza un estudio exploratorio y aplicado de mecanismos de cuatro barras (FBM), centrándose en las restricciones geométricas para un análisis avanzado que facilita la comprensión del diseño, la construcción y la cinemática, con el objetivo de aplicaciones prácticas en ingeniería y educación.

Problema: tradicionalmente, el diseño de FBM se ha basado en la teoría de mecanismos y máquinas, sin incorporar restricciones matemáticas geométricas avanzadas como la desigualdad de Hlawka. Esta omisión limita la capacidad de optimizar exhaustivamente estos sistemas para aplicaciones específicas.

Objetivo: desarrollar software en Matlab que integre criterios matemáticos basados en las desigualdades de Hlawka y Grashof, aplicando también el lagrangiano para el análisis cinemático y controladores PI, PD y PID para la dinámica de sistemas mediante FBM.

Metodología: para FBM se utilizó un enfoque integrado que combina análisis geométrico, cinemático y de control, desarrollando un algoritmo basado en la desigualdad de Hlawka. Esto se complementó con la creación de software en Matlab que ajusta los controladores según la ley de Grashof. La validación se realizó mediante comparaciones gráficas y análisis del error absoluto medio (EMA) con un caso práctico relevante.

Resultados: este proyecto destaca el potencial de un enfoque matemático innovador en el diseño de mecanismos de cuatro barras (FBM), enriqueciendo significativamente sus aplicaciones prácticas y formativas. Además, sienta un precedente importante para futuras investigaciones, proponiendo nuevas vías de estudio y exploración en cinemática avanzada y diseño mecatrónico, allanando así el camino para desarrollos innovadores en campos relacionados con la ingeniería y la educación técnica.

Conclusión: este proyecto establece una base para las aplicaciones de la síntesis FBM en implementaciones de control y acerca al usuario a la manipulación de parámetros y variables mediante software que garantiza la comprensión del fenómeno para el diseño y la construcción de dispositivos.

Originalidad: este estudio introduce un enfoque novedoso al integrar la desigualdad de Hlawka en el diseño de FBM, estableciendo un marco pionero para futuras investigaciones y desarrollos tecnológicos.

Limitaciones: las restricciones del estudio surgieron debido a la escasez de datos comparativos específicos disponibles, lo que subraya la necesidad de futuras investigaciones para una mayor evaluación.

Palabras clave: mecanismos de cuatro barras, desigualdad de Hlawka, ley de Grashof, software Matlab, controladores PID, diseño mecatrónico.

Resumo

Introdução: Esta pesquisa realiza um estudo aplicado e exploratório de mecanismos de quatro barras (FBM), com foco em restrições geométricas para uma análise avançada que facilite a compreensão do projeto, construção e cinemática, visando aplicações práticas em engenharia e educação.

Problema: Tradicionalmente, o projeto de FBM tem sido baseado na teoria de mecanismos e máquinas, sem incorporar restrições matemáticas geométricas avançadas, como a desigualdade de Hlawka. Essa omissão limita a capacidade de otimizar esses sistemas de forma abrangente para aplicações específicas.

Objetivo: Desenvolver software em Matlab que integre critérios matemáticos baseados nas desigualdades de Hlawka e Grashof, aplicando também o Lagrangiano para análise cinemática e controladores PI, PD e PID para dinâmica de sistemas usando FBM.

Metodologia: Uma abordagem integrada que combina análise geométrica, cinemática e de controle foi usada para FBM, desenvolvendo um algoritmo baseado na desigualdade de Hlawka. Foi complementado pela criação de software em Matlab que ajusta drivers de acordo com a lei de Grashof. A validação foi realizada usando comparações gráficas e análise de erro absoluto médio (MAE) com um estudo de caso relevante.

Resultados: Este projeto destaca o potencial de uma abordagem matemática inovadora no design de mecanismos de quatro barras (FBM), enriquecendo significativamente suas aplicações práticas e de treinamento. Além disso, ele estabelece um precedente importante para pesquisas futuras, propondo novas vias de estudo e exploração em cinemática avançada e design mecatrônico, abrindo caminho para desenvolvimentos inovadores em campos relacionados à engenharia e educação técnica.

Conclusão: Este projeto estabelece uma linha de base para aplicações de síntese de FBM em implementações de controle e aproxima o usuário da manipulação de parâmetros e variáveis por meio de software que garante a compreensão do fenômeno para o design e construção de dispositivos.

Originalidade: Este estudo introduz uma nova abordagem ao integrar a desigualdade de Hlawka no design de FBM, estabelecendo uma estrutura pioneira para pesquisas futuras e desenvolvimento tecnológico.

Limitações: Restrições do estudo surgiram da escassez de dados comparativos específicos disponíveis, ressaltando a necessidade de pesquisas futuras para avaliação posterior.

Palavras-chave: Mecanismos de quatro barras, desigualdade de Hlawka, lei de Grashof, software Matlab, controladores PID, projeto mecatrônico.

1. INTRODUCTION

Mechanisms have traditionally been defined as mechanical systems composed of bars or rigid bodies that are articulated through joints, giving rise to a system with different degrees of freedom—at least one [1], [2], [3]. Their most important characteristic is the ability to transform an input movement, whether linear or rotational, into an output movement in the form of correctly transformed trajectories for various engineering applications [4], [5], for example, machines or prototypes for automation and control in mechanical or electronic engineering.

In this context, the study of four-bar mechanisms (FBM, see Figure 1) has been fundamental, as understanding the kinematics and dynamics of these systems is crucial for innovations in design and application. Traditionally, the design of these mechanisms has followed well-established principles in the Theory of Mechanisms and Machines [6], which define the feasibility of continuous movements throughout a complete cycle. However, beyond the conventional approach to multibody mechanical systems, the incorporation of mathematical methods based on geometric exploration—such as the Hlawka inequality [7]—introduces a novel premise for analyzing and optimizing the synthesis of FBMs.

Therefore, this paper advances the field of FBMs by integrating, for the first time, the Hlawka inequality into its design. This mathematical approach is applied to constructible and simulatable configurations, facilitating the implementation of advanced mechatronic applications.

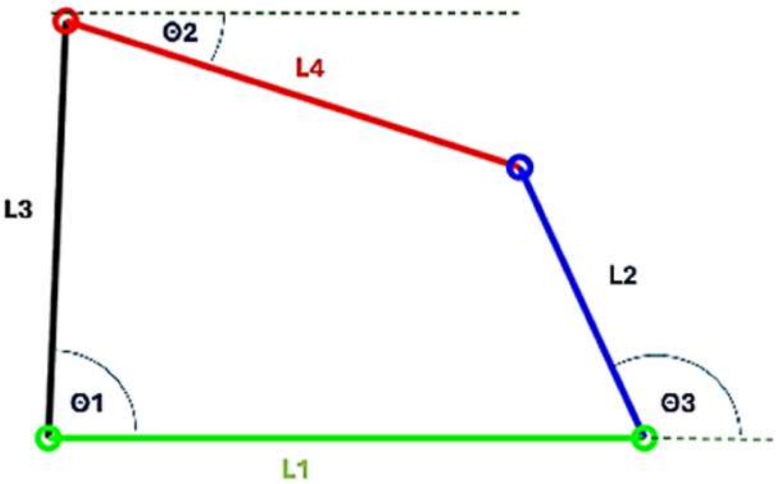

Figure 1. Four-bar articulated mechanism (FBM).
Source: Own work.

In addition to its theoretical relevance, this study develops an application in MATLAB© aimed at controlling these mechanisms [8], [9], [10], [11]. This novel approach not only validates the practical usefulness of the Hlawka inequality as a theoretical consideration in FBMs but also specifies the implementation of PID, PD, or PI controllers [12], [13], thereby facilitating dynamic simulation and real-time evaluation of the behavior of parameters considered in the kinematics and dynamics of FBMs.

Furthermore, this research can contribute to engineering education, as the study and analysis of mechanisms are fundamental components of undergraduate programs in Mechanical, Electronic, and Mechatronic Engineering, among others, worldwide—including in Colombia [14], [15]. Teaching and learning tools designed for advanced educational environments play a crucial role in helping students and professionals explore and understand the complexity of FBMs through interactive and didactic platforms, such as the one proposed in this study [16].

This article is structured as follows: first, the mathematical and notational elements central to the research are established. Next, a review of prior research on mathematical considerations in the design and implementation of software for FBMs is presented. This is followed by an outline of the materials and methodologies used, including a case study to contrast the obtained results. The discussion then addresses the results in terms of key parameters and variables. Finally, conclusions and potential future research directions are provided.

1.1. BACKGROUND AND NOTATION

In this document, vector spaces with an inner product are considered as a structure in a vector space V, where a function is defined as:: $\langle\ ,\ \rangle: V \times V \longrightarrow F$;

where F is a real or complex field. This function satisfies the following properties: 1. Linearity: $\langle \alpha x + \beta y, z\rangle = \alpha\langle x,z\rangle + \beta\langle y,z\rangle$ for all $x, y, z \in V$ $\alpha, \beta \in F$; 2. Symmetric commutativity: $\langle x, y\rangle = \overline{\langle y, x\rangle}$; 3. Positivity: : $\langle x, x\rangle \geq 0$; with equality if and only if $x = 0$. From this structure, natural geometric concepts such as the norm of vectors and the angle between them arise. The norm is defined as: $\| x \| = \sqrt{\langle x, x\rangle}$; and the cosine of the angle θ between two vectors x and y is given by $cos\ (\theta) = \frac{\langle x,y\rangle}{\|x\|\|y\|}$, [17].

The Hlawka or quadrilateral inequality is defined, in a vector space with inner product, considering three vectors (1):

$$\| x \| + \| y \| + \| z \| + \| x + y + z \| \geq \| x + y \| + \| y + z \| + \| x + z \| \quad (1)$$ [8]

This inequality indicates that the total length of all pairwise sums of three vectors is not greater than the perimeter of the quadrilateral formed by those vectors. In other words, a geometric criterion for constructing an FBM is that, given four sides, the sum of any three must not be less than or equal to the fourth.

Grashof's law characterizes the mobility of an FBM and is defined in terms of the lengths of the mechanism's links. (2) [8]:

$$s + l \leq p + q \tag{2}$$

Where:

s: Length of the shortest link, l: Length of the longest link and, p,q: Lengths of the two intermediate links

If the sum of the shortest and longest link lengths is less than or equal to the sum of the other two links, and the shortest link is adjacent to the longest, then the mechanism is capable of completing a full rotation.

According to equation (2), the FBM is classified based on the movement of the shortest link:

Crank-Rocker: The shortest link can rotate completely.

Double Crank (Drag Link): Both end links can rotate 360°.

Double Rocker: None of the links can complete a full rotation.

Regarding the analytical synthesis of the FBM, its kinematic implementation in digital form is facilitated by the Freudenstein equation, which serves as a geometric constraint. This equation is computed as the sum of the squares of (3) and (4): $L_2 Cos\theta_1 + L_4 Cos\theta_2 - L_2 Cos\theta_3 = L_1$ (3); (4); operationally, all angles are expressed in terms of a single angle through trigonometric treatment, [18], [19].

The equation of motion of the FBM, hereinafter, is established using the Lagrangian differential equation (5): $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mu}}\right) - \left(\frac{\partial L}{\partial \mu}\right) = \delta W$ (5); where μ is the generalized coordinate system, L is the Lagrangian and W is the work done by an external load. The Lagrangian is expressed as $L = T - U$; where T and U are the kinetic and potential energy of the FBM, respectively [20].

2. RESEARCH BACKGROUND REVIEW

As previously indicated, traditional mechanism analysis through computational tools [21]–[29], focusing on kinematic and dynamic analysis, has been conducted using robust and specialized proprietary CAD/CAE software such as ProEngineer, SolidWorks, SolidEdge, and ADAMS. Additionally, other proprietary mechanism analysis software, such as SAM [30] and Universal Mechanisms [31], has been utilized. Free alternatives like GIM [32] and Linkage [33] are also available and useful for designers. Meanwhile, open-source solutions oriented toward computational mechanics—such as Ch Mechanism Toolkit [34], Mesa Verde [35], and MBDyn [36]—offer flexibility but require users to have knowledge of both mechanisms and programming to adjust or modify them for research-specific needs.

However, given the orientation of this case study, a focused search and investigation were conducted over the past thirty years regarding the theoretical and mathematical considerations preceding the construction and applicability of software for FBM. This investigation revealed two primary elements that influenced the classification and study of relevant works: mathematical emphasis and user-end applicability. Consequently, these criteria were considered in the computational implementation design and its contrast with the case study.

With this central idea, studies have been identified regarding the kinematic analysis and design of four-bar mechanisms to simulate anthropomorphic movements. For example, one study analyzed human finger motion, where a CAD model was used to validate numerical results, presenting a comparative analysis between numerical and experimental kinematic data [37].

Alternatively, studies on configurable mechanism design for educational purposes have developed physical and mathematical models executed in MatLab©. One such study enabled students to interact with models, adjust parameters, and observe results in real time [38].

The kinematic and dynamic analysis of FBMs has also been applied to the design and manufacturing of outdoor exercise machines. This research compared results obtained through analytical mathematical modeling in MatLab© with multibody system simulations in ADAMS, achieving high precision [39].

Additionally, a study specifically on Hlawka's inequality explored its applicability in quadrilateral construction, drawing from works such as those of Ptolemy. While this study did not employ software for verification, it provided alternative theoretical foundations for practical applications in computational FBM implementations [40].

From a geometric perspective, the mobility analysis of four-bar planar mechanisms using a parallel coordinate system represents a significant advancement in understanding FBMs, as it facilitates intersection and angular movement analysis [41].

Meanwhile, the application of hybrid models—combining genetic algorithms and evolutionary strategies in the dimensional synthesis of mechanisms—has optimized trajectories and motion. This approach enables the identification of critical maximum or minimum points in initial or boundary conditions without requiring prior knowledge of the search space. As a result, it has proven effective in the optimal synthesis of FBMs, providing sets of solutions for different parameter configurations [42].

As previously mentioned, the development of educational tools in MatLab© for FBM kinematic analysis has enhanced the teaching of fundamental concepts. The inclusion of graphical user interfaces, allowing users to modify link lengths and evaluate compliance with criteria such as Grashof's law [43], adds significant value to the learning process.

From an experimental standpoint, the development of electromechanical teaching prototypes has enabled students to interact with FBMs in real time, integrating mechanical design, electronic control, and operational interfaces. These prototypes provide valuable hands-on and didactic experiences [44]. For example, hexapod robot designs illustrate the applicability of FBMs in robotics by combining simulation software with experimental observation to classify and analyze mechanism behavior [45].

Additionally, the development of Android applications for FBM kinematic studies has allowed these tools to be executed on mobile platforms, facilitating their use in both educational and professional fields. These applications enable the determination of key variables such as position, velocity, and acceleration, as well as the classification of mechanisms according to Grashof's law [46].

From a symbolic and geometric mathematical perspective, the use of GeoGebra for exploring and verifying quadrilateral inequalities has improved the visualization and understanding of these concepts. However, based on the research scope, the use of Hlawka's inequality in FBM applications remains largely unexplored [47]–[50].

3. MATERIALS AND METHODS

The implementation process was developed in the following phases:

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I. Mathematical Analysis of Four-Bar Mechanisms (FBM)

A comprehensive mathematical analysis of FBMs was conducted, incorporating Hlawka's inequality to evaluate and optimize possible configurations for their construction.

II. Characterization Using Grashof's Law

Building upon Phase I, Grashof's law was applied to determine the rotational characteristics of the FBM. Equations were formulated and solved to evaluate different configurations, ensuring they met the necessary conditions for continuous and efficient movement.

III. Mathematical Modeling and Numerical Verification

A mathematical model integrating Phases I and II was developed. Numerical simulations were performed to verify the accuracy of the model across various practical scenarios.

IV. MATLAB© Application Development

A computational tool was designed in MATLAB©, leveraging its capabilities for mathematical computation and dynamic simulation through Simulink. The solver **ode45** was employed for solving simultaneous differential equations at an appropriate sampling rate. The application synthesizes the equations and algorithms necessary for the kinematic analysis of FBMs while integrating expressions for **PID, PD, and PI controllers**.

V. Case Study and Comparative Analysis

A case study was selected based on previously documented research [18], [20]. This study analyzes a flexible four-bar mechanism dynamically using MATLAB© and Simulink. While the case study includes kinematic and dynamic analysis, it does not incorporate Hlawka's inequality in its geometric analysis. To quantify deviations between datasets, the **Mean Absolute Error (MAE)** metric was utilized.

VI. Data Extraction and Validation Using Automeris.io

The **automeris.io** tool [50] was employed to digitize and extract graph data from previous studies. This tool provided an efficient method for obtaining accurate numerical data from published graphs, which was essential for validation and comparison with the results produced by the developed software. Automeris.io's ability to convert graphical data into numerical form enabled a more detailed analysis and verification of mathematical models and simulations.

3.1. Software Features

3.1.1. Development Description

The development of this proposal was implemented in MATLAB© and Simulink. It features a graphical interface that includes input parameters, output parameters, a 2D simulation of the mechanism, and its classification.

Figure 2 illustrates the process, which is activated by pressing the "Play" button. This action initiates the simulation of the four-bar mechanism and captures relevant data for analysis.

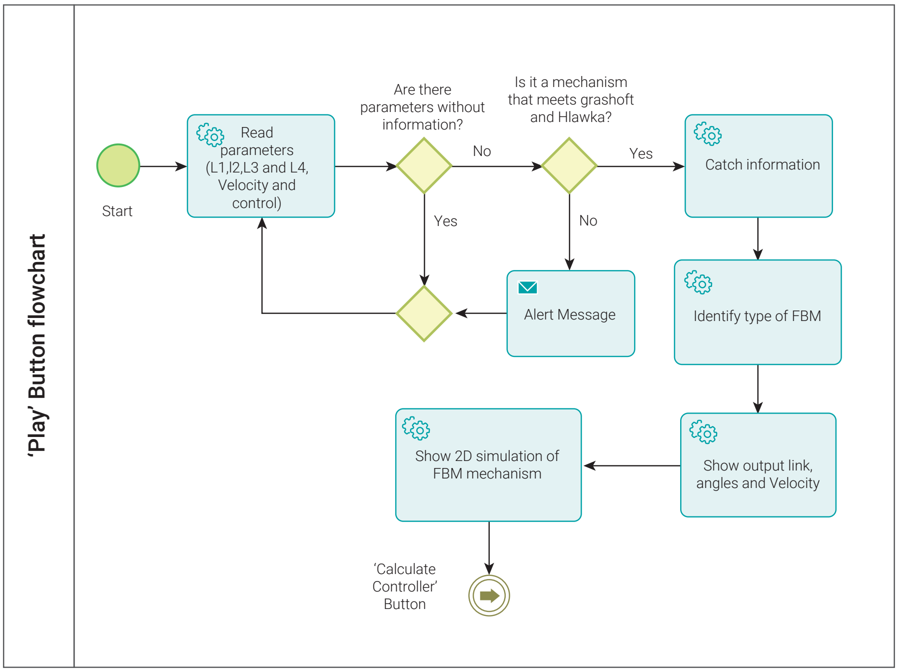

Figure 2. Play button flowchart.
Source: Own work.

On the other hand, Figure 3 illustrates the controller calculation process, which halts the simulation to process the information. This step generates the transfer function and produces the system's graphs with the applied controller.

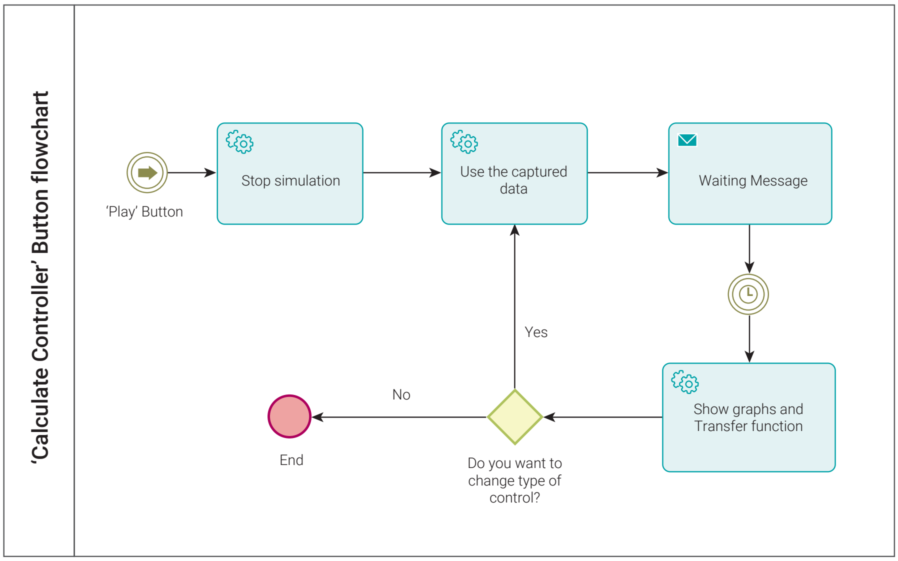

Figure 3. Calculate Controller button flowchart.
Source: Own work.

3.1.2. *User interface*

A user-friendly interface was developed, allowing users to easily enter parameters, run simulations, and visualize results in an intuitive way. The interface enhances interaction with the software, enabling real-time adjustments and analysis, as shown work in Figure 4.

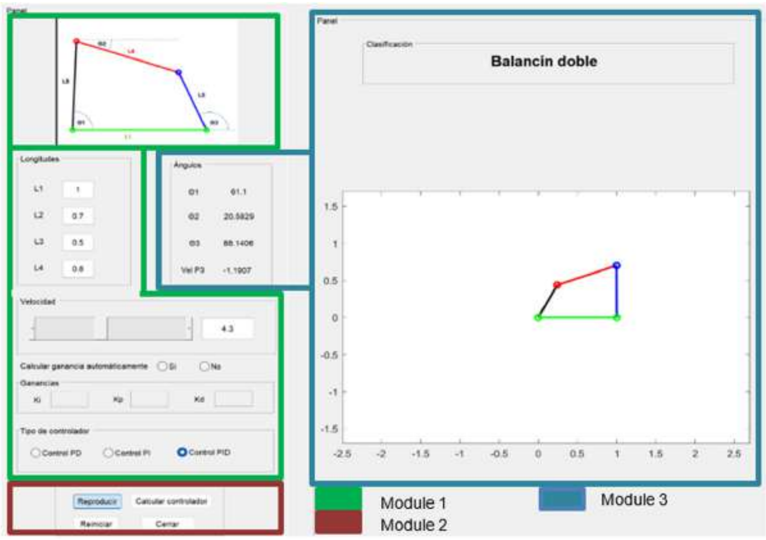

Figure 4. Graphical interface.
Source: Own work.

There are several modules within the interface, described as follows:

1. **Module 1: Input Data and Link Setup**

   This module corresponds to the convention of the links and angles of the mechanism to be simulated and is where the user must enter the following values:

   a) The lengths of the links L1, L2, L3, and L4.
   b) The velocity with which the simulation should be carried out.
   c) Whether or not to include Ki, Kp, and/or Kd gain values for the control calculation.
   d) The type of control to be applied (PI, PD, or PID).

2. **Module 2: Control Buttons**

   This module contains several control buttons:

   a) Play: Activated only when all the information in Module 1 is complete. Pressing this button starts the simulation and captures the input and output data of the mechanism, as well as other necessary data for the simulation.
   b) Calculate Controller: Stops the simulation and data capture, and proceeds with processing the information to obtain the transfer function and control behavior graphs.
   c) Reset: Clears all input data entered by the user.
   d) Close: Closes the application completely.

3. **Module 3: Results**

   In this module, users can view in real time:

   a) The angle and velocity of the output link (L3).
   b) A simulation of the movement of the mechanism.
   c) The type of mechanism being analyzed.

4. **Alert Messages:**

   There are two alert messages:

   a) Alert for Hlawka Condition Violation: Triggered when the link lengths entered by the user do not meet the Hlawka conditions.

Figure 5. Alert message.
Source: Own work.

b) **Calculate Controller:** Stops the simulation and data capture, and proceeds with processing the information to obtain the transfer function and control behavior graphs. When this button is pressed, a message is displayed asking the user to wait a few minutes while the control calculation is performed and the graph is displayed.

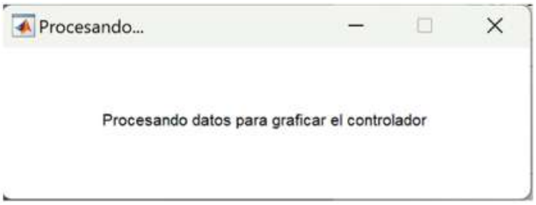

Figure 6. Waiting message.
Source: Own work.

4. RESULTS

Up next, we will provide a breakdown work of the results obtained throughout the research, including their interpretation and mathematical analysis, as well as their implementation in the software development process.

4.1 First Phase: Practical Validation of Hlawka.

In this first part, the viability of using the Hlawka inequality (1) for the construction of quadrilaterals is validated.

Three vectors, x, y and z, are defined in the plane, forming a quadrilateral together with the origin O. Point X is the endpoint of vector x, point Y is the endpoint of vector x+y, and point Z is the endpoint of vector x+y+z.

The vector interpretation of the defined system is carried out, applying the Triangular Inequality to the triangles formed by the sums of pairs of vectors

The sum of the lengths of the sides: $OX = \| x \|, XY = \| y \| XY, YZ = \| z \|,$ plus the length of the side $ZO = \| x + y + z \| \ is$:

$$OX = \| x \|, XY = \| y \| XY, \qquad YZ = \| z \|$$

- The sum of the lengths of the segments: $OY = \|x + y \|, XZ = \| x + z \| \ and$ $YZ = \| y + z \|$ is: $\| x + y \| + \| y + z \| + \| x + z \|$

According (1) $\| x \| + \| y \| + \| z \| + \| x + y + z \| \geq \| x + y \| + \| y + z \| + \| x + z \|$

This translates to the sum of the lengths of the sides, along with an additional "path" through the quadrilateral, being at least as large as the sum of the lengths of the segments connecting non-adjacent vertices, plus one of the sides.

4.2 Second Phase: Software development.

During the development of the project, software was created with a series of advanced functionalities that enabled detailed and accurate analysis of four-bar mechanisms (FBM). The capabilities of the developed software and its user interface are outlined below.

4.3 Third Phase: Control calculation.

Once it is confirmed that the proposed development meets the necessary criteria for modeling and simulating an FBM, the generation of the different PI, PD, and PID control models is carried out using the MatLab© tool. This allows for the creation of a transfer function and enables the comparison of the behavior of the FBM with and without control, as shown work in Figure 7.

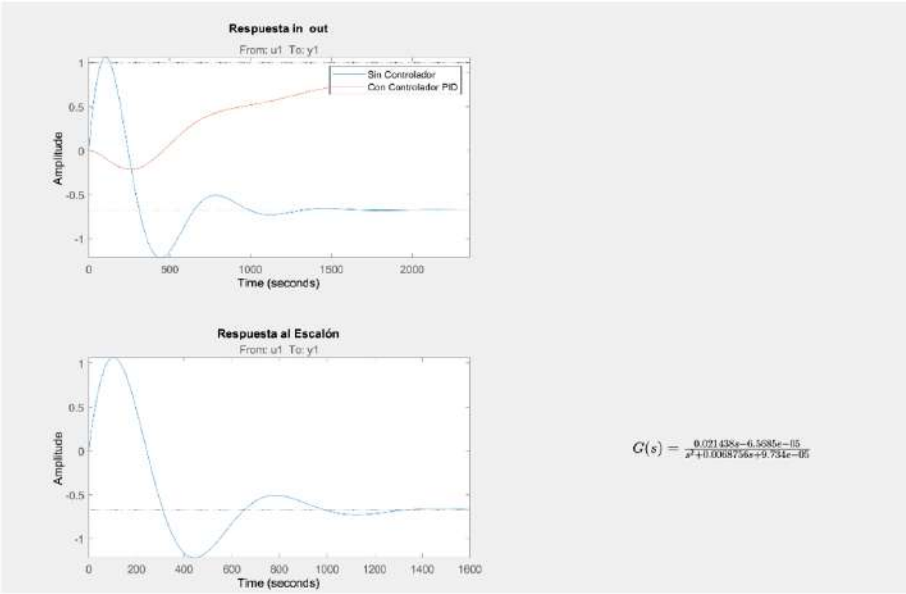

Figure 7. Control applied to the four-bar mechanism.
Source: Own work.

During the simulation, both input data (input angle of link L3) and output data (output angle of link L2) were recorded over time. As more data is collected or the capture time is extended, the precision of the results significantly increases, offering a more detailed view of the dynamic behavior of the system.

The quality of the simulation and the accuracy of the results depend largely on the chosen control function, whether it is a PI, PD, or PID controller. These controllers alter the system's response, and the resulting transfer function reflects the dynamic characteristics of the mechanism under different control conditions. This transfer function is crucial for analyzing and understanding how the system reacts to variations in the control parameters, enabling the optimization of the mechanism's performance. The detailed data from these simulations and their results are presented in Table 1.

Table 1. Data obtained in the simulation.

1	Time	Input Angle	Output Angle
2	6.4488304	-1.256637061	0.996978515
3	6.7742621	-1.308996939	0.908926356
4	7.0232718	-1.361356817	0.840773433
5	7.2731721	-1.413716694	0.783356303

(continúa)

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1	Time	Input Angle	Output Angle
6	7.5320475	-1.466076572	0.73312206
7	7.7878279	-1.518436449	0.688190031
8	8.0417127	-1.570796327	0.647398594
9	8.2985767	-1.623156204	0.609960026
10	8.5641472	-1.675516082	0.575305706
239	68.8816056	-4.398229715	-1.021896868
240	69.1215952	-4.345869837	-0.986179724
241	69.3544799	-4.29350996	-0.949595704
242	69.6095389	-4.241150082	-0.912247185
243	69.8623612	-4.188790205	-0.874221218
244	70.1154312	-4.136430327	-0.835592577
245	70.3669848	-4.08407045	-0.796426072
246	70.6210531	-4.031710572	-0.756778353
247	70.8706197	-3.979350695	-0.71669932
248	71.1244435	-3.926990817	-0.676233239

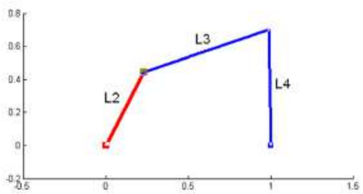
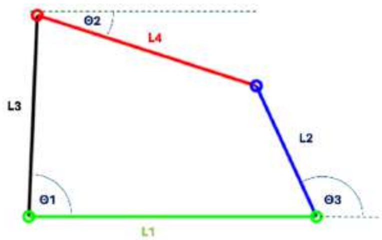
Source: Own work.

These results enable us to observe how the system adapts under different controller configurations, providing a crucial tool for both system analysis and design.

5. DISCUSSION

Initially, simulations of the implemented alternative are obtained, using the data recorded in [18] and [20] as references. These are presented below, with the conversion of links to those used in the proposal, as detailed in Table 2.

Table 2. Conversion of base article data and own work data.

Reference links	Measurements	Links of this proposal
 Figure 8. Link position reference image [25]		 Figure 9. Proposed image of link position Source: Own work

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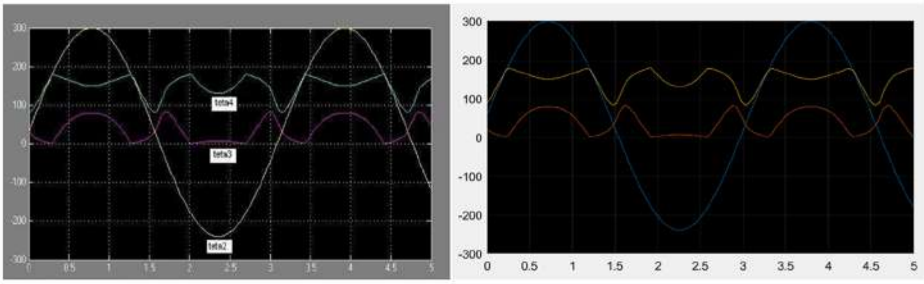
(viene)

Reference links	Measurements	Links of this proposal
L1	1	L1
L2	0.5	L3
L3	0.8	L4
L4	0.7	L2
θ_3	θ_2 (convention)	θ_2
θ_4	θ_3 (convention)	θ_3

Source: Own work.

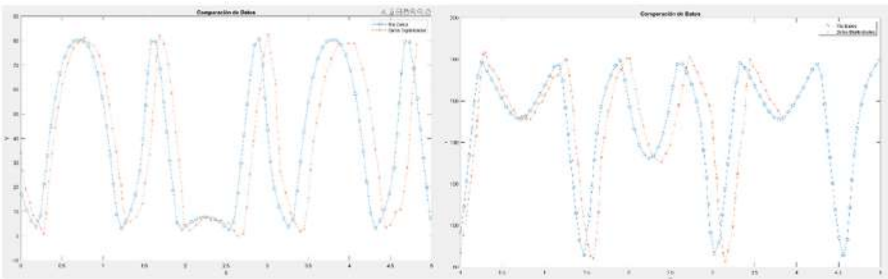
In [18] and [20], the generation of kinematic graphs for position, velocity, and acceleration over time is discussed; however, no specific data is provided to facilitate their generation. Using the automeris.io tool [50], the graph data from [18] and [20] were captured and compared with the data collected from the developed proposal, as shown work in Figures 10 to 15.

a. Displacement:



Displacement Graph reference document [25] Graph of the document under development

Figure 10. Comparison of displacement graphs over time.
Source: Own work.



θ_2 θ_3

Figure 11. Comparison of the points taken for the displacement with respect to time as θ_3 and θ_4 according to the reference document and respectively θ_2 and θ_3 for the purposes of the document in development.
Source: Own work.

In Figure 10, the displacement graphs of the links over time, as given in the reference documents, are shown work alongside the data produced by the developed program. The blue points in Figure 11 represent the data obtained from the developed program, while the red points represent the digitized data sourced from [18] and [20].

Upon analyzing the results in Figure 11, it can be observed that the points from both data sets are dispersed. However, they follow a similar trend and exhibit a comparable range of values, as indicated in Tables 3 and 4.

Table 3. Range of displacement data over time θ_2 and θ_3

Error calculation	θ_2		θ_3	
	Time	Degrees	Time	Degrees
Digitalized data	0.00 to 5.01	-0.06 to 82.75	0.00 to 4.99	82.41 to 183.27
Own work data	0.01 to 4.98	2.61 to 80.72	0.00 to 4.99	85.70 to 179.76

Source: Own work.

To quantify the difference between the data sets, the Mean Absolute Error (MAE) was calculated, generating the following results:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{y}_i]$$

Where:

n = is the number of observations.

y_i = is the observed value

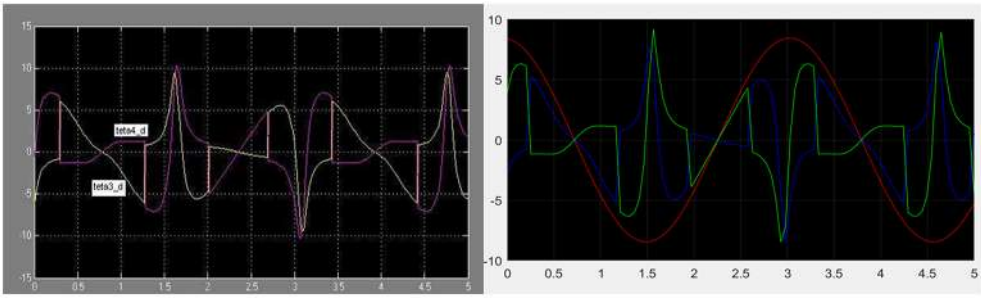
$\hat{y}_i$ = is the predicted or estimated value.

Table 4. Calculation of the error for displacement data over time θ_2 and θ_3.

Error calculation	Displacement	Displacement
MAE (Mean Absolute Error)	35.69	31.04

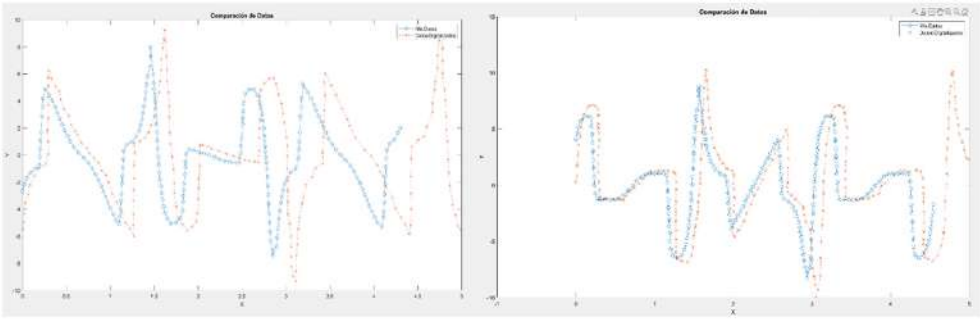
Source: Own work.

b. Velocity:



Velocity Graph reference document [25]   Graph of the document under development

Figure 12. Comparison of velocity graphs.
Source: Own work.



θ_2 θ_3

Figure 13. Comparison of the points taken for the velocities expressed as θ_3 and θ_4 according to the reference document and θ_2 and θ_3 respectively for the purposes of the document in development.
Source: Own work.

In Figure 12, the velocity graphs of the links with respect to time, taken from the reference document, are compared with the data produced by the developed program. The blue points in Figure 13 correspond to the data obtained from the developed program, while the red points represent the digitized data from the referenced thesis.

Upon analyzing the results in Figure 13, it can be observed that the points from both data sets are dispersed. However, they follow a similar trend, with the range of values detailed in Tables 5 and 6.

Table 5. Velocity data range θ_2 and θ_3.

Error calculation	θ_2		θ_3	
	Time	**Angular velocity**	**Time**	**Angular velocity**
Digitalized data	0.00 to 4.98	-8.40 to 8.03	0.00 to 4.99	82.41 to 183.27
Own work data	0.00 to 4.69	2.61 to 80.72	0.00 to 4.99	85.70 to 179.76

Source: Own work.

Table 6. Calculation of the error for Velocity data θ_2 and θ_3.

Error calculation	Velocity	Velocity
MAE (Mean Absolute Error)	4.19	31.04

Source: Own work.

c. Acceleration:

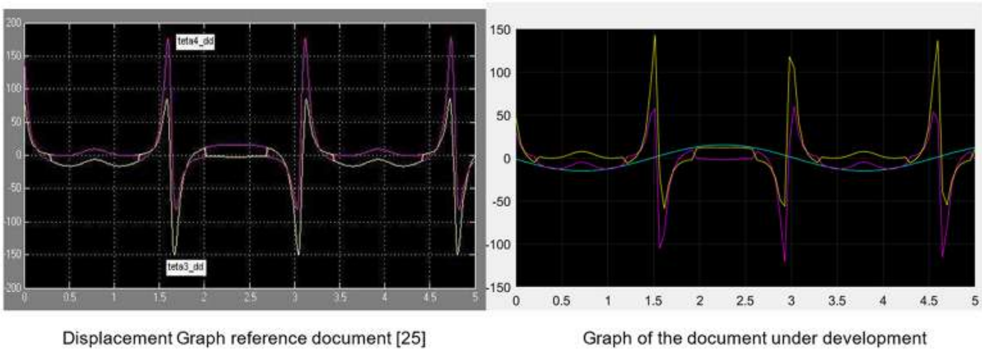


Figure 14. Comparison of acceleration graphs.
Source: Own work.

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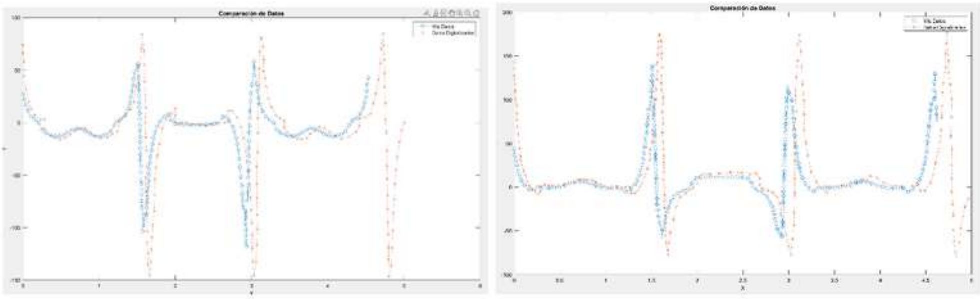

θ_2 θ_3

Figure 15. Comparison of the points taken for the accelerations expressed as θ_3 and θ_4 according to the reference document and respectively θ_2 and θ_3 for the purposes of the document in development.
Source: Own work.

Figure 14 presents the acceleration graphs of the links with respect to time, comparing the data from the reference document with the results produced by the developed program. The blue points in Figure 15 correspond to the data from the developed program, while the red points represent the digitized data from the referenced thesis.

Upon analyzing the results in Figure 15, it is evident that the points from both data sets are dispersed; however, they exhibit a similar trend. The range of data is outlined in Tables 7 and 8.

Table 7. Acceleration data range θ_2 and θ_3.

Error calculation	θ_2		θ_3	
	Time	Acceleration	Time	Acceleration
Digitalized data	0.00 to 5.00	-146.48 to 84.51	0.00 to 4.97	-79.58 to 178.17
Own work data	-117.77 to 58.04	-146.48 to 84.51	0.00 to 5.01	-56.14 to 139.13

Source: Own work.

Table 8. Error calculation for acceleration data θ_2 and θ_3.

Error calculation	θ_2	θ_3
MAE (Mean Absolute Error)	35.69	31.04

Source: Own work.

6. CONCLUSIONS.

Despite the numerical differences observed, the data ranges from both studies are highly similar, indicating that the initial conditions and parameters of the mechanism are well-aligned. The general trends in the graphs, comparing the original and the analyzed data, show a significant correspondence. This demonstrates that the dynamic behavior of the mechanism has been effectively captured, which is the primary objective of this case study: to validate that the developed program is comparable to other analyses conducted in this proposal, thus fulfilling the goal of modeling the movement of a four-bar mechanism.

The calculated error values provide a quantitative basis for evaluating the consistency between the results of this study and the data from [18] and [20]. As shown work in Figures 10 to 15 and Tables 3 to 8, although numerical differences exist, the dynamic behavior of the mechanism is captured in a similar manner in both studies.

Considering the similarity in the data ranges, the consistent general trends, and the acceptable error metrics, it can be concluded that the developed tool is both viable and valuable for analyzing and identifying the movement of FBMs. This method lays a solid foundation for future studies and applications in dynamic analysis, facilitating the understanding and optimization of complex mechanical systems.

In summary, the MatLab-based software developed for the analysis of four-bar mechanisms has proven to be an accurate and efficient tool. It integrates advanced algorithms that simulate the dynamic behavior of the mechanism, significantly improving the design process and reducing development time. The user-friendly interface makes it accessible to both engineers and students, widening its usability. The software's ability to implement and tune controllers, along with simulating different operating conditions, provides a flexible and adaptable platform. Furthermore, the inclusion of Hlawka's inequality and Grashof's law in the geometric verification enhances the robustness and reliability of the design.

The proposed development offers a reliable approach for studying the movement of FBMs, contributing to areas such as robotics with specific movements, mechanical advantages to amplify forces, velocities, or accelerations, the optimal conversion and control of various types of motion, and the kinematic analysis of complex mechanisms under adapted trajectories. These applications address ongoing challenges in both engineering and education.

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