Ingeniería Solidaria

New Approach and Integration for The Introduction of Dynamic Systems: Methodology, Models, And Relations

Nuevo enfoque y articulación para la introducción de sistemas dinamicos: metodología, modelos y relaciones

Nova abordagem e articulação para introdução de sistemas dinâmicos: metodologia, modelos e relacionamentos

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Received: December 5th, 2023
Accepted: March 11th, 2024
Available: May 7th, 2024

How to cite this article:
G. Ortiz Castillo, I.G. Perilla Martínez, A. Escobar Díaz, "New Approach and Integration for The Introduction of Dynamic Systems: Methodology, Models, And Relations," *Revista Ingeniería Solidaria,* vol. 20, no. 2, 2024.
doi: https://doi.org/10.16925/2357-6014.2024.02.04

Review article. https://doi.org/10.16925/2357-6014.2024.02.04

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Abstract

Introduction: This article is the result of the research "Analysis of the dynamic systems component of the engineering programs in control and automation and technology in industrial electronics", developed at the Francisco José de Caldas District University in the years 2022-2023.

Problem: The document addresses the lack of a clear and comprehensive structure in the analysis of linear dynamic systems, as well as the absence of an innovative approach to addressing these topics in traditional courses, which hinders their understanding and application in different disciplines.

Objective: The objective is to present a novel approach to analyzing linear dynamic systems, providing a complete framework of concepts, relationships, and important tools in this analysis, as well as a literature review based on over seventeen years of experience in courses and studies of this kind.

Methodology: A journey of concepts is proposed from signals and systems, through modeling with methods such as black box, white box, gray box, to linear analysis based on examples with calculation of differential equations, state representation, transfer function, and block diagrams.

Results: The document aggregates and articulates all the concepts of dynamic systems, along with the relationships and tools used, offering a more practical and intuitive approach to understanding the material.

Conclusion: The document provides a comprehensive and articulated view of key concepts in the analysis of linear dynamic systems, highlighting an innovative approach that facilitates their understanding. Its main contribution is to aggregate and articulate these concepts, along with the tools and relationships used, to offer a more practical and clear approach for their study.

Originality: The originality lies in proposing a novel and structured approach to analyzing linear dynamic systems, addressing the lack of clarity and completeness in traditional approaches.

Limitations: Although the document proposes a novel approach, it does not delve into specific aspects of some topics covered, which could limit the detailed understanding of certain concepts.

Deepening: The document could delve into the practical application of the proposed concepts and tools in real cases of dynamic systems, as well as into the comparison with traditional approaches to highlight the differences and advantages of the proposed new approach.

Keywords: Linear dynamic systems, articulation, methodologies, model relationships, modeling, dynamic analysis.

Resumen

Introducción: El artículo es producto de la investigación "Análisis del componente de sistemas dinámicos de los programas de ingeniería en control y automatización y tecnología en electrónica industrial", desarrollada en la Universidad Distrital Francisco José de Caldas en los años 2022-2023.

Problema: El documento aborda la falta de una estructura clara y completa en el análisis de sistemas dinámicos lineales, así como la carencia de un enfoque novedoso para abordar estos temas en cursos tradicionales, lo que dificulta su comprensión y aplicación en diferentes disciplinas.

Objetivo: El objetivo es presentar un enfoque novedoso para analizar sistemas dinámicos lineales, proporcionando una estructura completa de conceptos, relaciones y herramientas importantes en este análisis, así como una revisión bibliográfica basada en la experiencia acumulada en más de diecisiete años en cursos y estudio de este tipo de temas.

Metodología: Se propone un recorrido de conceptos desde señales y sistemas, pasando por modelización con métodos como caja negra, caja blanca, caja gris, hasta el análisis lineal con base en ejemplos con cálculo de ecuaciones diferenciales, representación de estado, función de transferencia y diagramas de bloques.

Resultados: El documento aglutina y articula todos los conceptos de sistemas dinámicos, junto con las relaciones y herramientas utilizadas, ofreciendo un enfoque más práctico e intuitivo para comprender el material.

Conclusión: El documento ofrece una visión integral y articulada de los conceptos clave en el análisis de sistemas dinámicos lineales, destacando un enfoque innovador que facilita su comprensión. Su principal contribución es aglutinar y articular estos conceptos, junto con las herramientas y relaciones utilizadas, para ofrecer un enfoque más práctico y claro para su estudio.

Originalidad: La originalidad radica en la propuesta de un enfoque novedoso y estructurado para analizar sistemas dinámicos lineales, abordando la falta de claridad y completitud en los enfoques tradicionales.

Limitantes: Aunque el documento propone un enfoque novedoso, no profundiza en aspectos específicos de algunos temas tratados, lo que podría limitar la comprensión detallada de ciertos conceptos.

Profundización: El documento podría profundizar en la aplicación práctica de los conceptos y herramientas propuestos en casos reales de sistemas dinámicos, así como en la comparación con enfoques tradicionales para resaltar las diferencias y ventajas del nuevo enfoque propuesto.

Palabras clave: Sistemas dinámicos lineales, articulación, metodologías, relación entre modelos, modelización, análisis dinámico.

Resumo

Introdução: O artigo é produto da pesquisa “Análise da componente de sistemas dinâmicos de programas de engenharia em controle e automação e tecnologia em eletrônica industrial", desenvolvida na Universidade Distrital Francisco José de Caldas nos anos 2022-2023.

Problema: O documento aborda a falta de uma estrutura clara e completa na análise de sistemas dinâmicos lineares, bem como a falta de uma abordagem inovadora para abordar esses tópicos em cursos tradicionais, o que dificulta sua compreensão e aplicação em diferentes disciplinas.

Objetivo: O objetivo é apresentar uma abordagem inovadora para análise de sistemas dinâmicos lineares fornecendo uma estrutura completa de conceitos relacionamentos e ferramentas importantes nesta análise bem como uma revisão bibliográfica baseada na experiência acumulada em mais de dezessete anos em cursos e estudo desses tipos de tópicos.

Metodologia: É proposta uma jornada de conceitos desde sinais e sistemas, passando por modelagem com métodos como caixa preta, caixa branca, caixa cinza, até análise linear baseada em exemplos com cálculo de equações diferenciais, representação de estado, função de transferência e diagramas de blocos.

Resultados: O documento reúne e articula todos os conceitos de sistemas dinâmicos, juntamente com os relacionamentos e ferramentas utilizadas, oferecendo uma abordagem mais prática e intuitiva para a compreensão do material.

Conclusão: O documento oferece uma visão abrangente e articulada dos conceitos-chave na análise de sistemas dinâmicos lineares, destacando uma abordagem inovadora que facilita a sua compreensão. Sua principal contribuição é reunir e articular esses conceitos, juntamente com as ferramentas e relações utilizadas, para oferecer uma abordagem mais prática e clara ao seu estudo.

Originalidade: A originalidade reside na proposta de uma abordagem nova e estruturada para analisar sistemas dinâmicos lineares, abordando a falta de clareza e completude nas abordagens tradicionais.

Limitações: Embora o documento proponha uma abordagem inovadora, ele não se aprofunda em aspectos específicos de alguns temas abordados, o que poderia limitar a compreensão detalhada de determinados conceitos.

Profundidade: O documento poderia aprofundar a aplicação prática dos conceitos e ferramentas propostas em casos reais de sistemas dinâmicos, bem como a comparação com abordagens tradicionais para destacar as diferenças e vantagens da nova abordagem proposta.

Palavras-chave: Sistemas dinâmicos lineares, articulação, metodologias, relacionamento entre modelos, modelagem, análise dinâmica.

1. INTRODUCTION

An analysis of dynamic systems is an approach used in various disciplines such as engineering, physics, economics, biology, ecology, etc., to understand and model systems that evolve and change over time. We could say that the world as we know it today would not be functioning without the proper use of this type of approach, as it forms the basis for solving and innovating in many problems in these disciplines. That's why, in the field of control engineering, it is a fundamental part of designing control systems.

A proper study of the systems to be controlled is required, as well as a good level of knowledge regarding the analysis and design of control systems. The preliminary study of classical methods for analysis and design from classical control theory (early 20th century principles) is essential, as it forms the basis for incorporating new techniques and procedures that have emerged due to advances in computing and electronics.

That is why courses in system dynamics dealing with modeling (developing mathematical models) and analysis of the response in such dynamic systems have become requirements in most engineering programs. Currently, there are numerous works by prestigious authors that provide students and professionals with a suitable means for understanding classical control techniques of systems; however, in many cases, it becomes complex, lacks order, context, and does not adapt in an organized manner to the current programs of different educational institutions, meaning it is not easy to understand.

The need for a text like this became evident in the face of these facts, and that is why it is focused on providing a general overview that gives context and a sequence of steps, without going into detail, which, in a first exposure to the subject, needs to be explained in detail. This article is intended for students and teachers in order to provide a more practical and intuitive approach to understanding the material.

Understanding and determining the dynamic response of a physical system (dynamic systems analysis) is a necessity for solving control engineering problems [21]. To do so, it requires two steps: system modeling and system response analysis (including system simulation) [3], [5], [13], [14], [15], [16], [17], [18], [19], [20].

Physical and dynamic systems represent the majority of cases in the field of engineering, as they are present in various industrial processes and in our daily lives. The concept of a system has different connotations and meanings. In general, it is a combination of interconnected components that interact with each other, with cause-and-effect relationships between input and output variables, to achieve a specific goal.

These systems are classified according to the context and their behavior. Within the context of their existence, they can be physical types (electrical [2], [3], [4], [5], [6], [15], [16], [19], [20], [22], [23], [24], [26], [29], [31], [32], [33], [34], [35], [41], [44], [48], [50], [51], [52], [53], [54], [55], [58], [59], [60], [61], [62], thermal, mechanical, hydraulic, pneumatic...) or abstract dynamic phenomena (economic, biological, transportation...).

Regarding their dynamic behavior, there are static and dynamic systems (those processes or objects that change over time).

To interact with a physical and dynamic system, it is important to understand how its inputs and outputs are related to each other in a simplified manner. This is achieved through the system model based on observed data, and it can take various forms: intuitive or mental models, graphical or non-parametric models, and mathematical, analytical, or parametric models [6], [7], [8], [9], [10], [11], [12]. The process of constructing this model is called modeling.

Henceforth, when referring to a "system," it will denote a physical and dynamic system, and when mentioning a "model," it will refer to a mathematical model.

In control engineering, a system or process can be viewed as any entity that performs a signal transformation. That is, it consists of input signals that can be manipulated (independent variables, reference signals, or disturbances) and output signals that can be observed (dependent or controlled variables). These are related through the system's transformation. This transformation, between input and output, is deciphered through the equations and parameters of the mathematical model.

The initial mathematical model is based on the fundamental physical laws that govern or describe the system's behavior. It can be a differential equation or a system of differential equations (dynamic equations) and is composed of functions, variables, and parameters. It can be constructed using the following methods: white-box method, when every internal component of the system is known (theoretical and carried out through mathematical modeling, contributing to time and resource savings); black-box method, when there is no information about the system's composition (empirical or experimental and carried out through process identification); and gray-box method, which is a combination of both white-box and black-box methods [1], [5], [10], [12], [13], [14], [21], [25], [32], [61].

Within the white-box method, there are two options to find the mathematical model: a mathematical procedure based on functional blocks or mathematical equations that describe the dynamic behavior of each component of the system, called mathematical modeling [2], [3], [4], [5], [6], [7], [14], [15], [16], [17], [18], [19], [20], [21], [23], [24], [27], [28], [29], [30], [31], [32], [35], [42], [47], [52], [53], [59], [61], [62], [63], [64]; and a graphical procedure based on the graphical representation of energy transfer between system components, called bond graphs or link diagrams. This visual modeling technique is used in engineering to represent multidomain physical systems.

Bond Graphs are based on representing the physical components of a system in terms of energy flows and are used in the analysis and design of complex dynamic systems. The main topics related to Bond Graphs include:

1. **Modeling of physical systems**: They are used to model multidomain physical systems, including mechanical, electrical, hydraulic, thermal, and other components. The modeling is based on the graphical representation of energy flows in different components, allowing for a more intuitive understanding of the system's behavior.
2. **System analysis:** Bond Graphs are used in system analysis to study the behavior of a system in response to different inputs and disturbances. The analysis is based on the graphical representation of energy flows in different components, providing a visual assessment of the system's response.
3. **System Design:** Bond Graphs are used in system design to assess the feasibility of different configurations and to optimize system performance. The design is based on the graphical representation of energy flows in different components, allowing for a more intuitive understanding of the interaction between various components.
4. **System Control:** Bond Graphs are used in system control to model and design controllers for complex dynamic systems. Control is based on the graphical representation of energy flows in different components, providing a more intuitive understanding of the system's interaction and response.

5. **System Simulation:** Bond Graphs are used in system simulation to model the behavior of complex dynamic systems. Simulation is based on the graphical representation of energy flows in different components, allowing for a more precise and detailed simulation of the system's behavior [22], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [65].

Both approaches have their own advantages and disadvantages and are useful in different situations depending on the nature of the system to be modeled.

Once the mathematical model is obtained, the next steps involve validating the model and analyzing the response. To achieve this, it is necessary to solve the mathematical model for the desired output. Finding the solution directly from the model in differential equations can be complex, tedious, and may not be feasible. Additionally, the controller design will depend on this solution.

That's why, since the early days of automatic control, new ways of representing the mathematical model have been developed. These are related to theories for the analysis and design of control systems that have emerged over time, in line with technological advances.

The most important representations of a mathematical model are:

1. **System of Differential Equations or Dynamics:** It is the foundational mathematical model, as mentioned before. It can be classified based on various criteria, including the order of the equation, whether it is linear or nonlinear, whether it is homogeneous or nonhomogeneous, among others. There are different methods to solve differential equations, including analytical methods and numerical methods. Analytical methods involve finding an exact solution to the differential equation (method of separation of variables, substitution method, method of undetermined coefficients, method of variation of parameters, Laplace transform method), while numerical methods rely on numerical approximations to obtain an approximate solution (Euler's method, improved Euler's method, Runge-Kutta method, and shooting method) [5], [15], [17], [21], [24], [25], [26], [27], [28], [30], [31], [32], [33], [34], [35], [36].
2. **Transfer Function:** It is a mathematical relationship between the Laplace transform of the system's output and the Laplace transform of the system's input, assuming zero initial conditions. It is a crucial tool in the analysis and design of control systems. It can be used to model physical and electromechanical systems, analyze the stability of a control system,

study the response of a control system, design controllers for control systems, analyze linear systems, and find the system's time response from its Laplace transform using the inverse Laplace transform [2], [3], [4], [5], [6], [13], [15], [16], [17], [18], [19], [20], [21], [23], [24], [25], [26], [31], [32], [33], [35], [47], [48], [49], [50], [51], [52], [53], [54], [55].

3. **State-Space System:** It is a mathematical model used to describe the dynamic behavior of a system. It consists of two first-order differential equations: the state equation and the output equation. The state equation describes the evolution of the system's internal state over time. This state is represented as a vector of state variables that describe the internal configuration of the system at a given moment. The state equation is a first-order differential equation that relates the rate of change of the state vector to the current state and the system's input. The output equation describes the relationship between the internal state of the system and the system's output. This equation relates the system's output to the internal state and the system's input. The output equation can be an algebraic or a differential equation, depending on the nature of the system. State-space representation is used to model complex dynamic systems such as electromechanical systems, control systems, communication systems, and biological systems. State-space models are useful for the analysis and design of systems as they allow simulation of the system over time, prediction of its future behavior, and optimization of its parameters. Solving state-space equations can be achieved using mathematical techniques such as Laplace transform, Fourier transform, and numerical solution of differential equations. Control techniques like feedback control and state estimation are also employed to control systems modeled by state-space. Topics related to state-space include system control, stability analysis, state observers, multivariable systems, and sensitivity analysis [2], [3], [4], [5], [15], [21], [22], [32], [33], [39], [48], [54], [59], [60].
4. **Block diagram:** It is a graphic representation of a control system that uses blocks to represent the different components of the system and the relationships between them. In a block diagram, the blocks represent input and output variables, processing elements, control systems, and the interactions between them. In a control system, the block diagram is used to illustrate how input signals are converted to output signals through the various elements of the system. The blocks can represent elements such as amplifiers, filters, adders, integrators and derivatives, among others.

The block diagram is a useful tool for the design and analysis of control systems, since it allows you to visualize the interactions between the different components of the system. It is also used for simulation and mathematical analysis of control systems, since it allows the equations and relationships that describe the behavior of the system to be clearly represented. Some of the benefits of using block diagrams in control systems are: It allows you to clearly visualize the operation of the system, it facilitates the identification of possible problems and areas for improvement in the system, it facilitates the understanding of the relationships between the components of the system, and facilitates the analysis and mathematical simulation of the system. Topics related to block diagrams in control systems include control system design, stability analysis, transient response analysis, multivariable system control, and discrete-time control systems [2], [3], [4], [5], [13], [14], [15], [16], [17], [19], [20], [21], [23], [24], [32], [33], [34], [35], [52], [53], [54], [61].

5. **Signal Flow Diagram**: Signal flow diagrams are a valuable tool in control theory used to represent and describe the dynamics of dynamic systems and design controllers. They are also employed to depict communication systems and signal processing [2], [3], [5], [17], [20], [21], [33], [35], [48], [53].

There are three theories commonly used for the analysis and design of control systems: classical control theory, modern control theory, and robust control theory. Each of them has its own techniques and tools for controller design and is applied in different situations, depending on the characteristics of the system to be controlled and the associated uncertainties [2], [3], [4], [5].

2. METHODOLOGY

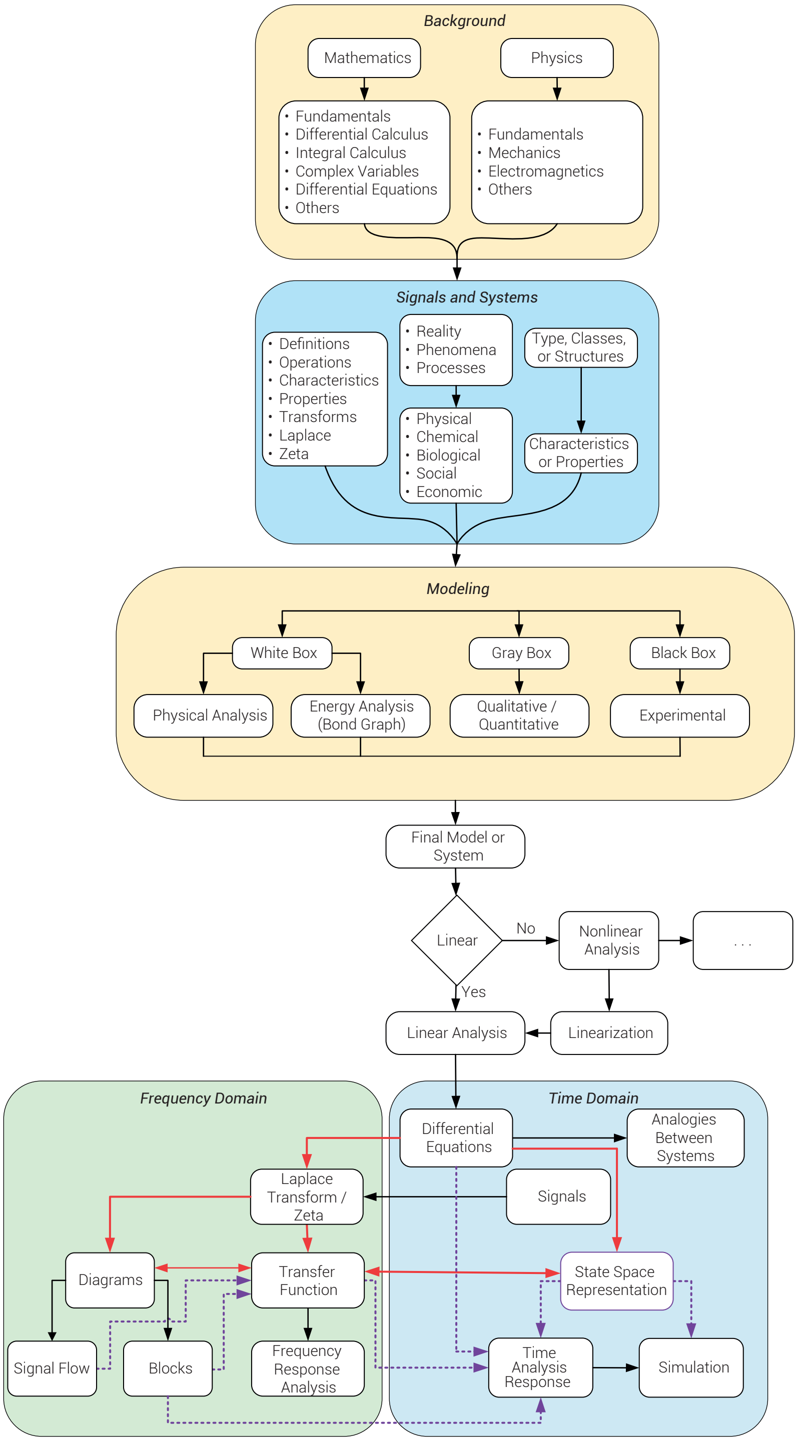

Figure 2-1. Model Representation

Source: own work

To model and analyze dynamic systems, a background or prerequisite knowledge is required. This involves studying signals, systems (especially types and classes), their properties and characteristics, as well as their structures and relationships, along with differential equations, basic engineering courses, mathematics, and physics. When these foundations are in place, there are three methodologies for approaching the modeling of a system or phenomenon, which are: white-box, gray-box, and black-box.

White-box modeling involves analyzing, from a physical, chemical, or biological perspective, a phenomenon based on pre-established laws in these branches of science and deriving a mathematical model based on observed facts. Additionally, an energy analysis can be conducted using the Bond Graph methodology to formulate this mathematical model.

A significant portion of the modeling and study of dynamic systems is based on this type of white-box modeling. In courses, in theory, and in books, a great deal is invested in this type of modeling. In it, the nature of a phenomenon, whether it is physical, chemical, biological, or of any other kind (economic, social, spatial, astrophysical, microbiological, quantum, etc.), is considered. Any of these phenomena has been studied by some branch of science, which has defined some minimum laws governing these phenomena. In this way, the expert conducting the analysis of the phenomenon establishes a relationship and deduces a minimum set of equations that model said phenomenon.

The black-box methodology is based on experimentation. Based on a phenomenon under analysis, suitable stimuli are designed for this phenomenon, and responses are measured. Using the stimulus signals and the response signal, a mathematical model representing the dynamic behavior of the phenomenon under analysis is deduced. This black-box or experimental methodology follows some basic steps:

First: The most important step is the design of the experiment, which involves planning in great detail the stimulus that will be applied to the phenomenon. This stimulus must be configured and planned specifically for that phenomenon, in such a way that the main objective is to obtain the best information through the responses obtained from the phenomenon or system being analyzed.

Second: Selecting the structure can be, in linear models, a transfer function or a state representation, both in continuous and discrete time. For the transfer function or state representation, it can also be a fuzzy inference system or a neural network. The mathematical basis of the structures we can choose from is broad in theory. The selection of the structure depends on the phenomenon or system we are analyzing. Each structure fits better depending on the behavior exhibited by the phenomenon.

Black-box or experimental modeling involves the following steps: experiment design, structure selection, parameter calculation, and validation.

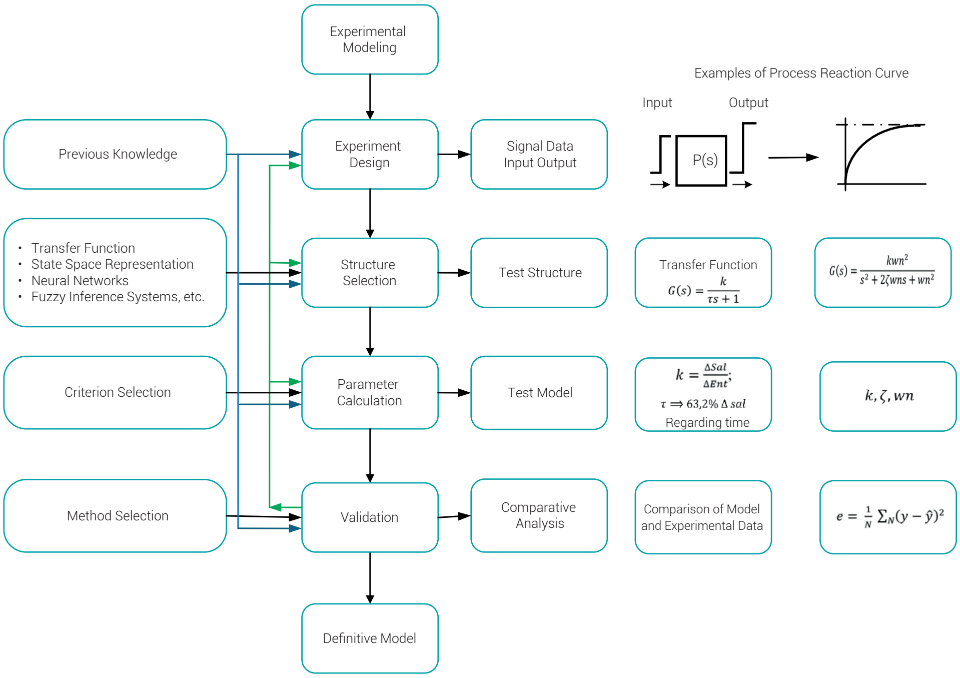

Figure 2-2. Experimental model
Source: own work

At the end of these steps, the model, structure, or set of equations representing the physical phenomenon under analysis is concluded.

- **Design of the experiment:** Prior knowledge of the phenomenon under analysis is required to design, in sufficient detail, the stimulus to be applied to the phenomenon or system. This ensures that complete information is obtained from the responses to the stimuli. Based on the input data, considering the input signals and output signals, the mathematical model is then concluded.
- **Selection of the structure:** Structures can be diverse; the mathematical foundation is extensive. They can be transfer functions, state representations, fuzzy inference systems, neural networks; they can have linear or non-linear connotations. Nowadays, structures are varied. The selection of the structure depends on how well it fits the behavior of the phenomenon being analyzed. Therefore, depending on the results and the designer's experience, the structure is concluded.

- **Parameter Calculation:** Parameterization is carried out by defining the chosen structure. The selected structure has a set of parameters. These parameters must be calculated in this step. Generally, an optimization process is carried out to calculate the parameters that best fit the experiment being conducted. As a basis for this least squares optimization process, the parameters are calculated through an optimization process. In such a way, the response of the model under test must be adjusted to approximate the response that produces the least error with respect to the relevant experimental data.
- **Validation:** It consists of comparing the experiment data, the data of the response of the phenomenon being analyzed, with the model, structure, or set of equations that are under test. The same input that was applied to the phenomenon or system is applied to the structure under test, a response is obtained, and the model under test is compared with the experiment. If there is an appropriate similarity, that model is chosen to represent the phenomenon or system; if not, it is discarded, and any of the previous steps are repeated.

The gray box modeling methodology involves combining qualitative and quantitative methods to describe the dynamic behavior of the phenomenon or system under analysis. A particular example for describing phenomena, like the gray box method, is fuzzy inference systems. In this case, through natural language, we can provide a description of behaviors and translate them into a mathematical model.

After following a modeling methodology, the result of this process is to have a set of equations representing the phenomenon under analysis. This set of equations can be linear, non-linear, differential, algebraic, and may exist in a continuous or discrete context. For example, they can also be difference equations.

In this document, emphasis is placed on the modeling and analysis of linear dynamic systems. If necessary, a linearization process must be applied to the represented equations to arrive at a set of linear equations and initiate the process.

Starting from the minimum set of linear equations, which must include differential equations, a connection can be made with other structures. From these equations, transfer functions, state representations, and block and flow diagrams can be defined. These structures also allow for the representation of the system. The relationships in this type of structure are unambiguous. If you have the differential equation, you can jump to the transfer function, state representation, or any of the diagrams. If you have the state representation, you can jump to the equations, transfer function, and diagrams. It means that if you have one of the structures, you can easily represent the others.

On the other hand, it is worth noting that in the time domain, there are the models, equation structures, and state representation. In the frequency domain, there are transfer functions, block diagrams, and flow diagrams.

Particularly, block diagrams are used for simulation, while flow diagrams are mainly employed for calculating transfer functions. In classical control theory, transfer functions are predominantly used, whereas in modern control theory, state representations are more commonly utilized.

In the frequency domain, analysis through the Laplace transform is crucial, as it allows us to describe transfer functions, flow diagrams, and block diagrams. Frequency analysis can only be performed under the transfer function structure. It's important to remember that transforms, like Laplace, are only defined for signals, and through this tool, the solution or calculation of the solution in time for differential equations, transfer functions, and state representations can be achieved. Of course, before calculating the solution, stimuli and initial conditions must be defined.

The most commonly used structure for simulation is the block diagram, but simulation can also be applied to equations, depending on the group of initial equations, as well as to the transfer function and state representation. It is also important to remember the analogy between systems, which allows representation through an electrical circuit of any other system, whether it be thermal, mechanical, etc. As a final result of the analogy, we have the set of equations representing the phenomenon we are analyzing and a set of equations for an electrical circuit that are analogous to the initial model but functionally operate in the same way.

3. LINEAR ANALYSIS: DYNAMIC ANALYSIS WITH AN ELECTRICAL CIRCUIT

In this section, the methodology explained in Section 2 is described and implemented through the modeling and analysis of a dynamic system, specifically by modeling and dynamically analyzing an electrical circuit. This methodology is applicable to any dynamic system under analysis.

In Figure 3-1, the electrical circuit is presented. The first step is to identify the nature of the phenomenon in this context, which is an electrical circuit. The laws of physics and electronics that describe it are applied, such as Ohm's law, Kirchhoff's laws, mesh analysis, nodal analysis, and Thévenin's and Norton's laws. All these laws apply to circuit analysis and are grouped together to integrate them and deduce the equations that represent the dynamic behavior of the system.

Next, the inputs and outputs are identified. In Figure 3-2, the input variables, output variables, and state variables are described. In the circuit, there are two input variables (the sources), two state variables (the capacitor voltage and the coil current), and two outputs. By identifying the number of state variables, it is determined how many differential equations must be found to represent the dynamic model. Since there are two state variables, either a second-order differential equation or two first-order differential equations must be found. Additionally, since two outputs were defined, two output equations must be found. In conclusion, based on the analysis and data presented earlier, two first-order differential equations and two output equations must be obtained to represent the dynamic behavior of the system. These equations can only depend on the input variables and state variables.

After having the set of laws of physics and electronics that apply to the modeling of electrical systems, identifying input and output variables and state variables, an analysis is conducted on the circuit under consideration. The equations representing its dynamic behavior are deduced. It begins with nodal analysis, integrates mesh analysis, and applies Ohm's analysis to conclude the equations.

3.1. Differential Equations

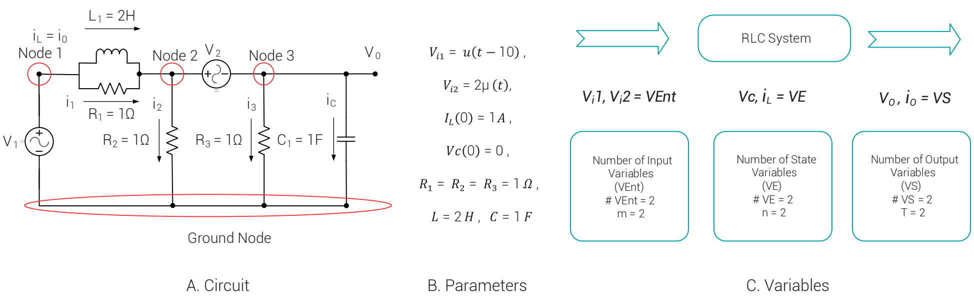

Figure 3 1. Circuit RLC
Source: own work

Starting from the general diagram for the analysis of electrical circuits in Figure 2.1, the Input Variables (VEnt), State Variables (VE), and Output Variables (VS) are identified on the circuit (Figure 3-1).

The equations can only depend on Input Variables (VEnt) and State Variables (VE), that is, for this case, V_i1, V_i2, V_C, I_L. The dynamic analysis is performed, the phenomenon is analyzed, and a relationship is established between the known equations. As a result of the above, the equations that model the behavior of the RLC circuit are:

$$V_o = V_c \ , \quad I_o = I_L \tag{1}$$

Equation 1:

$$\frac{dVc}{dt} = \frac{1}{C}\left(-V_c\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)+I_L+\frac{V_{i1}}{R_1}-V_{i2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right) \tag{2}$$

Equation 2:

$$\frac{dI_L}{dt} = \frac{1}{L}\left(V_{i1}-\left(V_{i2}+V_c\right)\right) \tag{3}$$

3.2. Transfer function

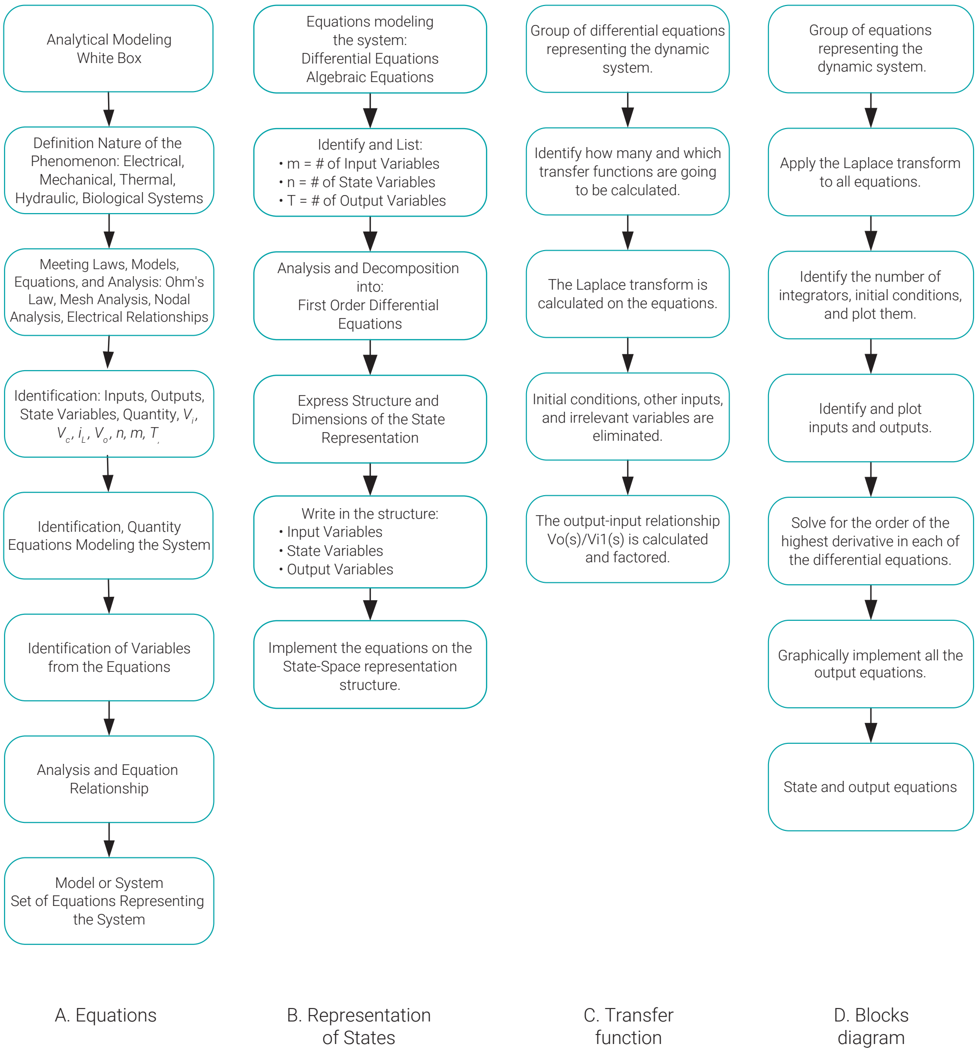

Figure 3 - 2. Calculation Methodologies
Source: own work

Based on the identification of inputs and outputs of the system, it can be determined how many transfer functions can be calculated from the equations representing the dynamic behavior of the system. In this case, there are four transfer functions, each output related to each of the inputs. It is important to note that the definition of a transfer function is the Laplace transform of an output signal in relation to the Laplace transform of an input signal, with initial conditions set to zero and other inputs set to zero.

For example, if you want to calculate the transfer function of the output voltage Vo with respect to the voltage of Source 1, Source 2 is set to zero, and all initial conditions from dynamic analysis are eliminated. In this way, when performing the Laplace transform, there are no initial conditions or other inputs. With the resulting equations, the variable of the coil current must be eliminated so that all equations are in terms of the capacitor voltage and thus represent the output voltage in the transfer function. This methodology can be followed for the calculation of the remaining three transfer functions, which are as follows in total.

It is worth remembering that through the transfer function, the stability of a system can also be expressed more quickly. The characteristic polynomial, which is the denominator polynomial of the transfer function, is taken, set to zero, and its roots are calculated. These roots are the poles of the system, and for the system to be stable, the real part of the poles must be negative. As observed, for all transfer functions of this system, all characteristic polynomials of the four transfer functions are the same. Moreover, they must be the same, which means that all transfer functions have the same poles; therefore, the system is stable.

It is also possible to create the pole-zero plot, as shown in the following figures. Zeros are the roots of the polynomials in the numerator of all transfer functions, and in the plots, poles are represented as crosses, while zeros are represented as circles. In this way, in the left half-plane of the Laplace plane, the stability region of the system can be analyzed. If all poles are in the left half-plane of Laplace, the system is stable, as all poles have a negative real part.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}}{\mathcal{L}} \frac{\{Y(s)\}}{\{U(s)\}} \Bigg| \begin{array}{l} \text{Condiciones Iniciales} = 0 \\ \text{Otras Entradas} = 0 \end{array}$$

First: We have the differential equations that model the behavior of the system RLC:

Second: Two inputs and two outputs of the system are evident, resulting in four different configurations for the transfer function, which are:

$$(1)\frac{V_o(s)}{V_{i1}(s)}, \quad (2)\frac{V_o(s)}{V_{i2}(s)}, \quad (3)\frac{I_o(s)}{V_{i1}(s)}, \quad (4)\frac{I_o(s)}{V_{i2}(s)} \tag{4}$$

Transfer Function $(1)\,\frac{V_o(s)}{V_{i1}(s)}$, initial conditions are assumed = 0, and the input not involved in this case $V_i2 = 0$ and it is eliminated Vc y I_L, Vc = Vo:

The transfer function $(1)\,\frac{V_o(s)}{V_{i1}(s)}$ is equal to:

$$\frac{V_o(s)}{V_{i1}(s)} = \left[\frac{(R_1 + Ls)(R_2R_3)}{R_1R_2R_3LCs^2 + LsR_2R_3 + LsR_1R_3 + LsR_1R_2 + R_1R_2R_3}\right] \tag{5}$$

The transfer function is obtained:

$$\frac{V_o(s)}{V_{i1}(s)} = \left[\frac{2s + 1}{2s^2 + 6s + 1}\right] \tag{6}$$

The Pole-Zero plot of the transfer function is obtained in the S-plane.

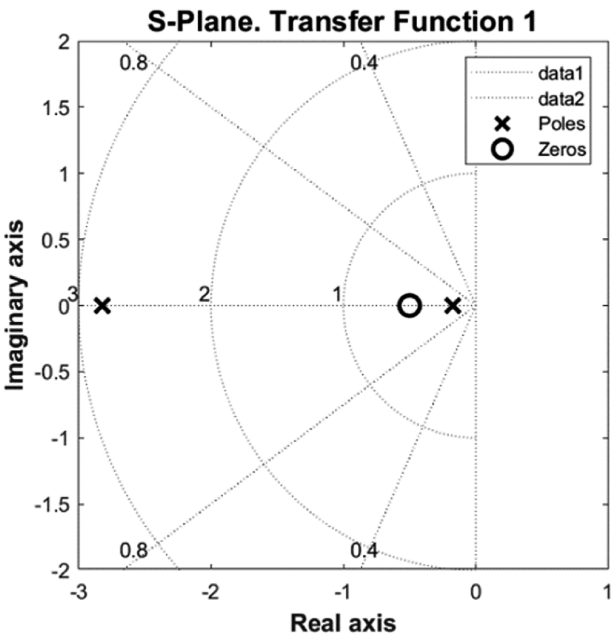

Figure 3-3. S-Plane (Poles and Zeros) Transfer Function 1.
Source: own work

Transfer Function $(2)\frac{V_o(s)}{V_{i2}(s)}$, initial conditions are assumed = 0, and the input not involved in this case $V_{i1} = 0$ and it is eliminated Vc y I_L, Vo = Vc:

The transfer function $(2)\frac{V_o(s)}{V_{i2}(s)}$ is equal to:

$$\frac{V_o(s)}{V_{i2}(s)} = \left[\frac{-(R_1R_2 + R_2Ls + R_1Ls)(R_3)}{R_1R_2R_3LCs^2 + LsR_2R_3 + LsR_1R_3 + LsR_1R_2 + R_1R_2R_3}\right] \quad (7)$$

The transfer function is obtained:

$$\frac{V_o(s)}{V_{i2}(s)} = \left[\frac{-4s - 1}{2s^2 + 6s + 1}\right] \quad (8)$$

The Pole-Zero plot of the transfer function in the S-plane is obtained.

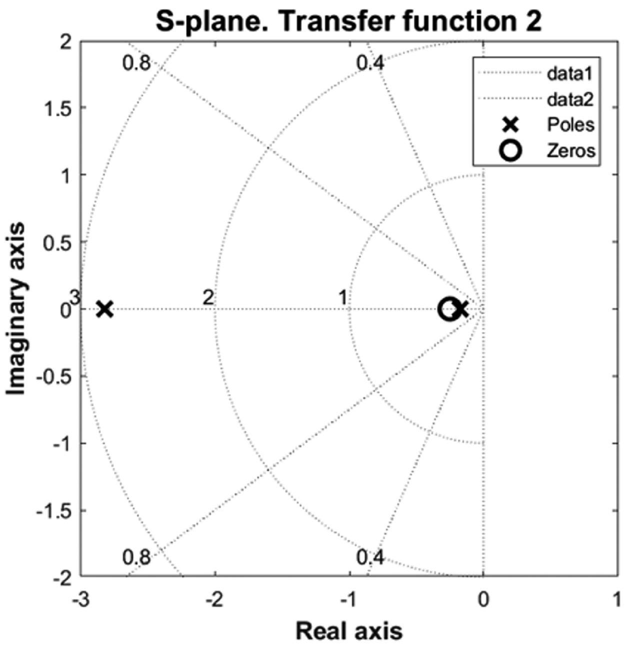

Figure 3-4. S-Plane (Poles and Zeros)
Transfer Function 2.
Source: own work

Transfer Function $(3)\frac{I_o(s)}{V_{i1}(s)}$, Initial conditions are assumed = 0, and the input not involved in this case $V_{i2} = 0$ and is eliminated Vc y I_L = Io:

The transfer function $(3)\,\frac{I_o(s)}{V_{i1}(s)}$ is equal to:

$$\frac{I_o(s)}{V_{i1}(s)} = \left[\frac{(R_2 + R_3 + R_2R_3Cs)(R_1)}{R_1R_2R_3LCs^2 + LsR_2R_3 + LsR_1R_3 + LsR_1R_2 + R_1R_2R_3}\right] \quad (9)$$

The transfer function is obtained:

$$\frac{I_o(s)}{V_{i1}(s)} = \left[\frac{s + 2}{2s^2 + 6s + 1}\right] \quad (10)$$

The pole-zero plot of the transfer function in the S-plane is obtained.

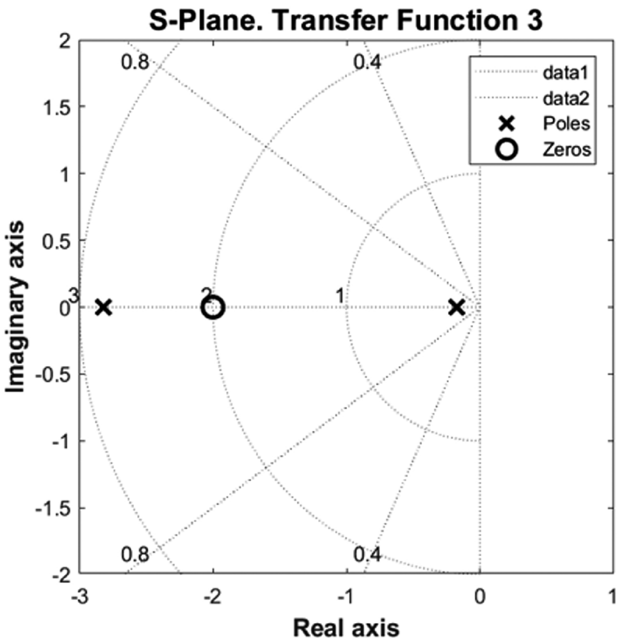

Figure 3-5. S-Plane (Poles and Zeros) Transfer Function 3.
Source: own work

Transfer Function $(4)\,\frac{I_o(s)}{V_{i2}(s)}$, Initial conditions are assumed = 0, and the input not involved in this case $Vi2 = 0$ and is eliminated Vc y I_L, $I_L = Io$:

The transfer function $(4)\,\frac{I_o(s)}{V_{i2}(s)}$ is equal to:

$$\frac{I_o(s)}{V_{i2}(s)} = \left[\frac{-(1+R_3Cs)(R_1R_2)}{R_1R_2R_3LCs^2 + LsR_2R_3 + LsR_1R_3 + LsR_1R_2 + R_1R_2R_3}\right] \quad (11)$$

The transfer function is obtained:

$$\frac{I_o(s)}{V_{i2}(s)} = \left[\frac{-s-1}{2s^2+6s+1}\right] \quad (12)$$

The pole-zero plot of the transfer function in the S-plane is obtained.

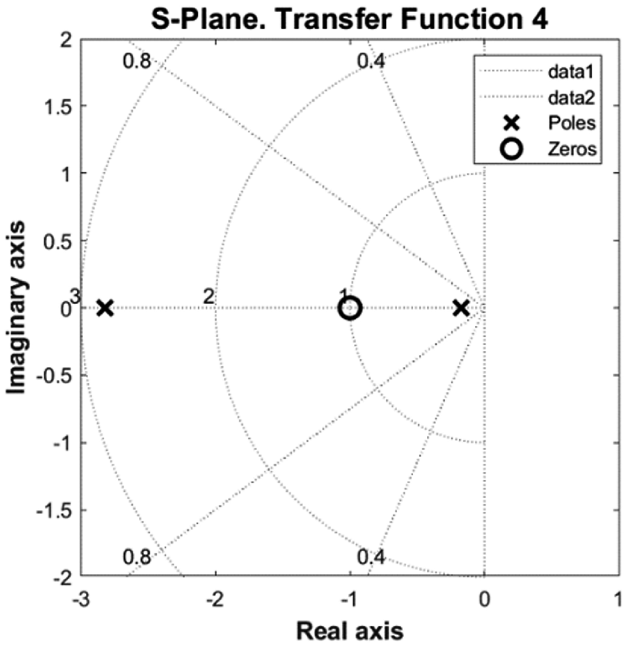

Figure 3-6. S-Plane (Poles and Zeros) Transfer Function 4.
Source: own work

3.3. Block Diagram

The block diagram is the graphical representation of the differential equations and output equations that represent the dynamic behavior of the system. This diagram has the advantage of being able to represent both linear and non-linear systems. The block diagram is in the frequency domain, in the Laplace domain. To construct block diagrams, the following methodology is followed:

The Laplace transform is calculated over the entire set of equations that represent the dynamic behavior of the system.

The orders of the differential equations represent the number of integrators within the block diagram. If a differential equation is of second order, there are two integrators in the block diagram. If in the set of equations there are two first-order differential equations, there is one integrator associated with each variable of the corresponding differential equation. For example, if there are two second-order differential equations, then there would be four integrators within the block diagram.

Identify the input and output variables. Take the equations and solve for the highest derivatives of the differential equations. Then proceed to create the plot. First, place the integrators that are present within the block diagram. For this example, there are two integrators, one associated with the differential equation of the capacitor voltage and another associated with the differential of the coil current. In each integrator, the variable and its derivative, and the initial condition, are related. On the right side of the diagram, place the system's output, and on the left side, place the system's input variables. Once the integrators relating the variables and their derivatives are in place, and the inputs and outputs are on the plot, implement the differential equation. The result for the analyzed dynamic system, for the set of equations of the analyzed dynamic system, is as follows:

The block diagram is the graphical representation of the differential equations and output equations that represent the dynamic behavior of the system. This diagram has the advantage of being able to represent both linear and non-linear systems. The block diagram is in the frequency domain, in the Laplace domain. To build block diagrams, the following methodology is followed:

The Laplace transform is calculated over the entire set of equations that represent the dynamic behavior of the system.

The orders of the differential equations represent the number of integrators within the block diagram. If a differential equation is of second order, there are two integrators in the block diagram. If there are two first-order differential equations in the set, there is one integrator associated with each variable of the corresponding differential equation. For example, if there are two second-order differential equations, there would be four integrators within the block diagram.

The variables of input and output are identified. The equations are taken, and the highest derivatives of the differential equations are solved. Then, the graph is created. Firstly, the integrators present within the block diagram are placed. For this example, there are two integrators: one associated with the differential equation of the capacitor voltage and another associated with the derivative of the coil current. In each integrator, the variable and its derivative, along with the initial condition, are related. On the right side of the diagram, the system outputs are placed, and on the left side, the input variables of the system are placed. Once the integrators relating the variables and their derivatives are placed, and the inputs and outputs are positioned in the graph, the differential equation is implemented. The result on the analyzed dynamic system, based on the set of equations of the analyzed dynamic system, is as follows:

First: All the equations (-) and their Laplace transforms must be obtained.

Second: Implement all algebraic and output equations.

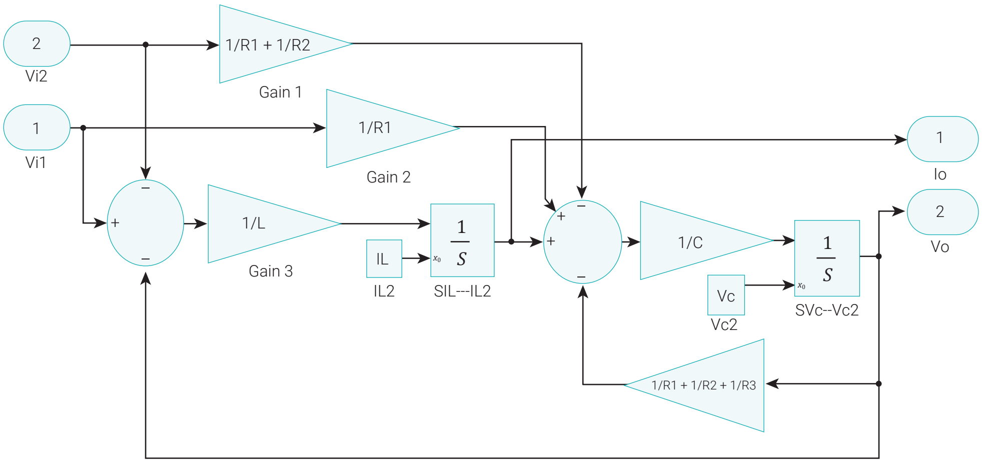

Figure 3-7. Block Diagram of the System RLC
Source: own work

3.4. State Space Representation

The state representation is the most commonly used model structure in the modeling, analysis, and control of advanced systems, also known as modern control. The state representation model implies that, through a mathematical relationship, the set of state variables and the input can calculate any future state of the system. Structurally, the state representation consists of two equations: the state equation and the output equation. The elements of these two equations are matrices, and their dimensions depend on the number of inputs, outputs, and state variables that the system under analysis has.

To build the state representation from the set of equations representing the dynamic behavior of the system, the following steps are followed:

1. If there are equations present in the representation, and if there are second-order equations in the set of equations, they must be decomposed into first-order differential equations.
2. Subsequently, the number of inputs, state variables, and outputs are ratified and identified again.
3. Next, the highest derivatives present in the equations are solved.
4. The structure is built; As the two equations that represent the state-space structure are matrices that depend on the dimensions of the inputs, outputs, and state variables, their dimensions are known and can be incorporated into that structure.

In the state equations, there are the state variable matrix and the matrix of the first derivatives of the state variables. The input matrix and the output variable matrix are also placed in these equations. The rest of the matrices (A, B, C, and D) are matrices with constant elements. In this sense, the next step is to place the first derivatives of the state variables in the matrix of the first derivatives of the state variables. Then, the input variables are placed in the input variable matrix, and finally, the output variables are placed in the output matrix.

The last step is to implement the equations into the structure. The differential equations are taken and implemented into the state equation. Then, the output equations of the system are implemented into the matrix of the output equation of the state representation.

For this exercise, considering the system we are analyzing, which has two state variables, two inputs, and two outputs, the final representation is as follows. The state representation is, by definition, the minimum set of equations that, in relation to the state variables and system inputs, allow defining the dynamic behavior of the system at any future point in its state.

Structurally, the state representation has two equations: the state equation and the output equation. All variables within the equations are matrices. Matrices a, b, c, and d are matrices with constant elements. Matrix 'x' and the state variable matrix ˙X˙ are the matrix of the first derivatives of the state variables. Matrix 'u' is the input matrix of the system, and Matrix 'y' is the matrix of the system's output variables. The dimensions of all matrices are defined by the number of state variables, the number of inputs, and the number of outputs defined in the system.

To build the state representation from the dynamic equations that represent a system, the following steps are followed: identify the number and state variables, identify the number and input variables, and identify the number and output variables.

If there are higher-order differential equations, the next step is to decompose them into first-order differential equations. For example, if there are second-order differential equations, they should be decomposed into two first-order differential equations. Then, express the structure of the state representation and design the matrices with their dimensions. After this, place the variables in the state variable matrix, in the matrix of the first derivatives of the state variables, in the input matrix, and in the output matrix. Finally, implement the equations.

For our exercise, we have two first-order differential equations, two output equations, which means the number of state variables is two, the number of inputs is two, and the number of outputs is two. It is not necessary to decompose differential equations. Therefore, the state representation is as follows:

The structural characteristics are as follows:

State Equation	$\dot{X} = A \cdot X + B \cdot U$	$\dot{X}_{nx1}$ X_{nx1} Y_{Tx1} U_{mx1}
State Equation	$Y = C \cdot X + D \cdot U$	A_{nxn} B_{nxm} C_{Txn} D_{Txn}

Characteristics: A, B, C, D are constant matrices, X = Matrix of State Variables, $\dot{X}$ = Matrix of Derivatives of State Variables, Y = Matrix of Output Variables, U = Matrix of Input Variables, Constant Matrix: Whose elements are fixed values.

First: The equations modeling the RLC system must be obtained, in order to solve for the higher derivatives in the differential equations.

Second: Determine the value of m, n, T.

Third: Obtain the State Space Representation equations that model the system RLC:

State Equation:

$$\begin{bmatrix} \dot{V}_C \\ \dot{I}_L \end{bmatrix}_{2x1} = \begin{bmatrix} -\frac{1}{c}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) & \frac{1}{c} \\ -\frac{1}{L} & 0 \end{bmatrix}_{2x2} \cdot \begin{bmatrix} V_C \\ I_L \end{bmatrix}_{2x1} + \begin{bmatrix} \frac{1}{CR_1} & -\frac{1}{c}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \\ \frac{1}{L} & -\frac{1}{L} \end{bmatrix}_{2x2} \cdot \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix}_{2x1} \quad (13)$$

Output Equation:

$$\begin{bmatrix} V_o \\ I_o \end{bmatrix}_{2x1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2x2} \cdot \begin{bmatrix} V_C \\ I_L \end{bmatrix}_{2x1} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2x2} \cdot \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix}_{2x1} \quad (14)$$

3.5. Time Domain Analysis

Time domain analysis can be performed on both the set of equations representing dynamic behavior and on the transfer function, state-space representation, and block diagram. Specifically, the block diagram is more commonly used in simulation. Of course, to calculate time responses, it is necessary to know the parameter values, such as the resistance of capacitors and inductors, as well as the values of stimuli, in this case, the signals from the sources. Based on these values, the responses for each of the models are calculated.

For the set of differential equations, the Laplace transform is calculated, initial conditions are replaced, and non-important variables are eliminated. For example, in the analysis, the inductor current is eliminated, and the transform of the sources is replaced, removing the capacitor voltage, leaving only the equations in terms of the necessary variable, in this case, the inductor current (output current). This procedure is carried out as follows:

In the case of transfer functions, the transfer function is taken, and only the input being analyzed is considered to calculate the corresponding output. In this case, the responses of each transfer function are calculated based on the analyzed stimulus.

The simulation of the block diagram can be carried out in simulation software such as Matlab or any other similar tool. In this way, the mathematical model constructed for the block diagram can be implemented in Matlab, and the result is presented with the following characteristics.

3.5.1. Solution of Differential Equation

3.5.1.1. Complete Solution

With the system equations to calculate V_0

First: Have the differential equations that model the system's behavior RLC.

Second: Have the values of the inputs, initial conditions, and constants of the system:

Third: Replace R_1=R_2=R_3 ,L,C, in the equations.

Fourth: Apply Laplace transform to the equations:

$$sV_o(s) = -3V_o(s) + I_o(s) + V_{i1}(s) - 2V_{i2}(s) \tag{15}$$

e-ISSN 2357-6014 / Vol. 20, no. 2 / may-august 2024 / Bogotá D.C., Colombia
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$$sI_o(s) - I_o(0) = \frac{1}{2}\left(V_{i1}(s) - V_{i2}(s) - V_o(s)\right) \tag{16}$$

Grouping terms, we get:

$$V_o(s)\left[\frac{2s^2 + 6s + 1}{2s}\right] = V_{i1}(s)\left[\frac{2s + 1}{2s}\right] - V_{i2}(s)\left[\frac{4s + 1}{2s}\right] + \frac{2}{2s} \tag{17}$$

Solving for $V_o(s)$ the equation is obtained:

$$V_o(s) = \frac{\left(2 + V_{i1}(s)(2s + 1) - V_{i2}(s)(4s + 1)\right)}{[2s^2 + 6s + 1]} \tag{18}$$

Fifth: A sectional analysis of the equation is performed to obtain the sectional time response and then sum it.

$$\begin{aligned} V_o(s) &= V_{o1}(s) + V_{o2}(s) - V_{o3}(s) \\ V_o(s) &= \frac{2}{[2s^2 + 6s + 1]} + \frac{V_{i1}(s)(2s + 1)}{[2s^2 + 6s + 1]} - \frac{V_{i2}(s)(4s + 1)}{[2s^2 + 6s + 1]} \end{aligned} \tag{19}$$

From the equations $V_{o1}(s)$, $V_{o2}(s)$, $V_{o3}(s)$:
The time response of is obtained:

$$V_{o1}(t) = 0.3779e^{-(0.1771)t}u(t) - 0.3779e^{-(2.8228)t}u(t) \tag{20}$$

$$V_{o2}(t) = -0.6891e^{-(0.1771)(t-10)}u(t-10) - 0.3110e^{-(2.8228)(t-10)}u(t-10) + 1u(t-10) \tag{21}$$

$$V_{o3}(t) = 0.6223\, e^{-(0.1771)(t)}u(t) + 1.3779e^{-(2.8228)(t)}u(t) - 2u(t) \tag{22}$$

The total time response Vo is represented in the following graph.

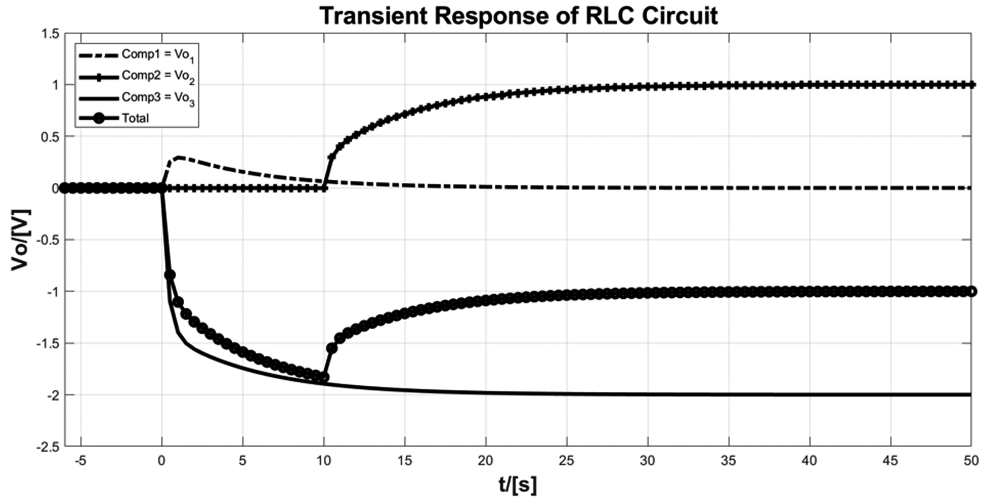

Figure 3-8. System Output Vo.
Source: own work

3.5.2. Solution with the Block Diagram

Next, the different configurations of the circuit conditions and their respective time responses are observed.

Condition 1: The time response for the variables of the equation is:

$V_{i1} = u(t-10)$,

$V_{i2} = 2$,

$I_L(0) = 1\ A$,

$Vc(0) = 0$,

$R_1 = R_2 = R_3 = 1\ \Omega$,

$L = 2\ H$,

$C = 1\ F$

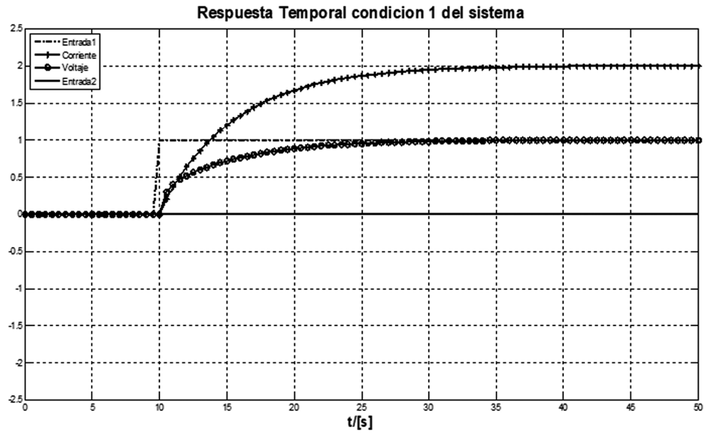

Figure 3 9. Temporal Response Condition 1
Source: own work

Condition 2: The temporal response for the variables of the equation is:

$V_{i1} = 0$,

$V_{i2} = 2$,

$I_L(0) = 1A$,

$Vc(0) = 0$,

$R_1 = R_2 = R_3 = 1\Omega$,

$L = 2H$,

$C = 1F$

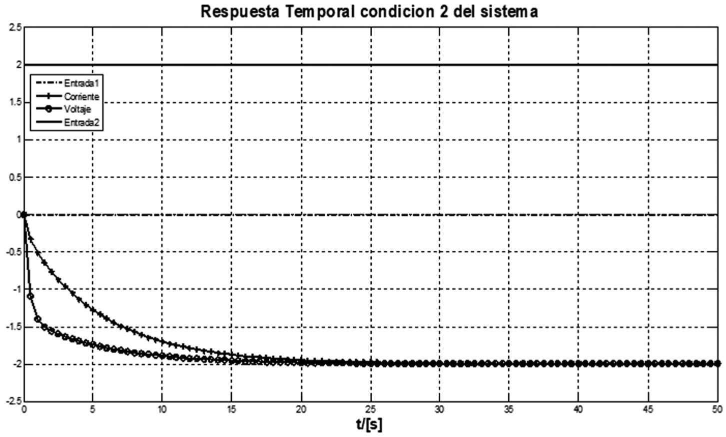

Figure 3-10. Temporal Response for Condition 2
Source: own work

Condition 3: The temporal response for the variables of the equation is:

$V_{i1} = 0$,

$V_{i2} = 0$,

$I_L(0) = 1A$,

$Vc(0) = 0$,

$R_1 = R_2 = R_3 = 1\Omega$,

$L = 2H$,

$C = 1F$

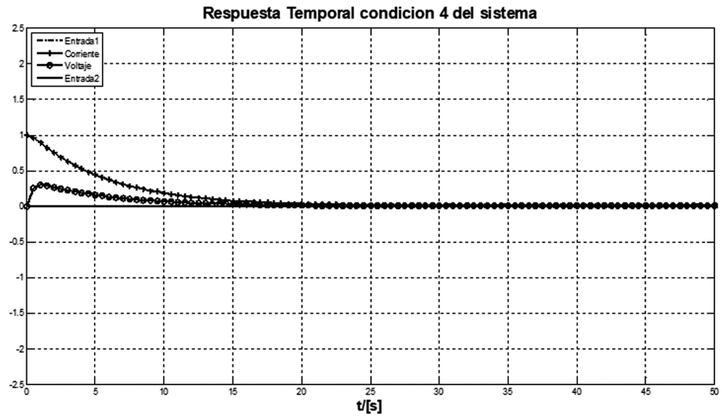

Figure 3-11. Temporal Response for Condition 3
Source: own work

The total response for the circuit conditions of the equation is:

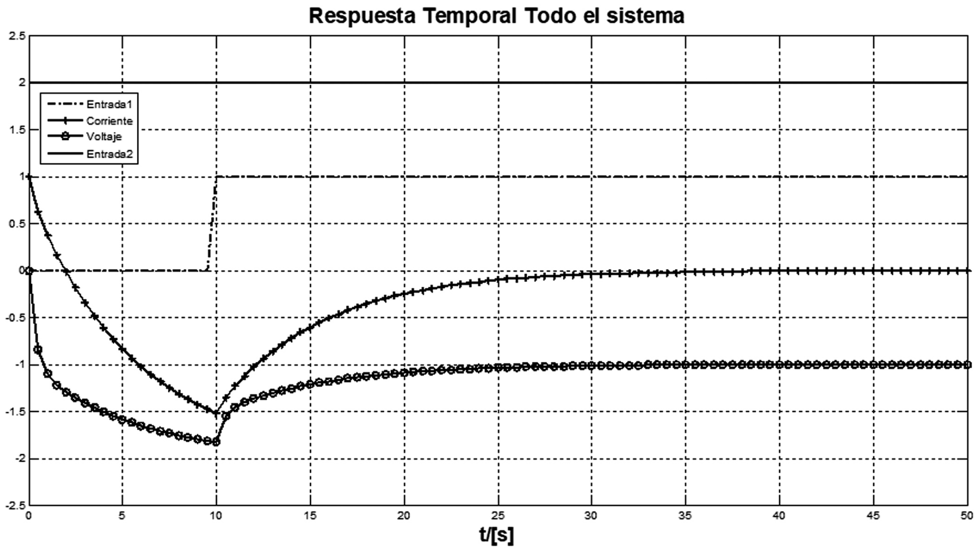

Figure 3-12. Temporal Response
Source: own work

4. CONCLUSIONS

In conclusion, this article provides a clear and detailed methodology for the introduction of dynamic systems, focusing on the analysis and calculation of the response of an RLC electrical circuit. The use of concept maps and flowcharts as comprehension tools proves to be very helpful for readers. Additionally, an overview of dynamic systems analysis is presented, emphasizing linear and time-invariant systems. The article also addresses different types of systems and models, as well as modeling methods to obtain a mathematical model of a system. In summary, this article serves as a comprehensive and practical guide for those who wish to learn about the analysis of dynamic systems and apply it in their area of study or work.

This provides an overview of all the concepts and topics that are important when analyzing and modeling dynamic systems. A simple procedure for black-box modeling is presented, especially in physical systems. A methodology is introduced for calculating the transfer function, creating block diagrams, and constructing the state representation from equations that describe the system's behavior. It is clear that the transfer function can also be calculated from the state representation. Importantly, the block diagram can be reduced to calculate the transfer function. In fact, if any of the structural model forms are known, one can arrive at each of them. If the state representation and all transfer functions are known, the system's representing equations can be derived, and vice versa.

All relationships between the models are also presented; the relationships that can exist between each of the models. From the differential equations, the state representation, transfer functions, block diagrams, flowcharts can be calculated, and with any of them, one can arrive at the others.

5. RECOGNITION

We mainly want to thank the Francisco José de Caldas District University for providing the scenarios to study and analyze dynamic systems, in the XUE research group and in the BARION hotbed. We also thank the students of the dynamic systems and control systems groups who contributed to the construction of the concepts, procedures and models presented in this document.

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