Hidden Markov Models for early detection of cardiovascular diseases

Modelos de Markov ocultos para la detección temprana de enfermedades cardiovasculares

Modelos ocultos de Markov para detecção precoce de doenças cardiovasculares

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Abstract

Introduction: This article, developed between 2022 and 2023 within the framework of Applied Stochastic Processes by the SciBas group at the Universidad Distrital Francisco José de Caldas, focuses on the role of Hidden Markov Models (HMM) in predicting cardiovascular diseases.

Problem: The addressed issue is the need to enhance the early detection of heart diseases, emphasizing how HMM can address uncertainty in clinical data and detect complex patterns.

Objective: To evaluate the use of Hidden Markov Models (HMM) in the analysis of electrocardiograms (ECG) for the early detection of cardiovascular diseases.

Methodology: The methodology comprises a literature review concerning the relationship between HMM and cardiovascular diseases, followed by the application of HMM to prevent heart attacks and address uncertainty in clinical data.

Results: The findings indicate that HMM is effective in preventing heart diseases, yet its effectiveness is contingent upon data quality. These results are promising but not universally applicable.

Conclusions: In summary, this study underscores the utility of HMM in early infarction detection and its statistical approach in medicine. It is emphasized that HMM is not infallible and should be complemented with other clinical options and assessment methods in real-world situations.

Originality: This work stands out for its statistical and probabilistic approach in the application of Hidden Markov Models (HMM) in medical analysis, offering an innovative perspective and enhancing the understanding of their utility in the field of medicine.

Limitations: It is recognized that there are limitations, such as dependence on data quality and variable applicability in clinical cases. These limitations should be considered in the context of their implementation in medical practice.

Key words: Stochastic processes, Markov chains, hidden Markov chains, ECG.

Resumen

Introducción: Este artículo, desarrollado entre 2022 y 2023 en el marco de Procesos Estocásticos Aplicados por el grupo SciBas de la Universidad Distrital Francisco José de Caldas, se enfoca en el papel de las cadenas de Markov ocultas (HMM) en la predicción de enfermedades cardiovasculares.

Problema: El problema abordado es la necesidad de mejorar la detección temprana de enfermedades cardíacas, y se destaca cómo las HMM pueden abordar la incertidumbre en los datos clínicos y detectar patrones complejos.

Objetivo: Evaluar el uso de modelos de Markov ocultos (HMM) en el análisis de electrocardiogramas (ECG) para la detección temprana de enfermedades cardiovasculares.

Metodología: La metodología incluye una revisión de la literatura sobre la relación entre las HMM y las enfermedades cardiovasculares, seguida de la aplicación de HMM para prevenir infartos y abordar la incertidumbre en los datos clínicos.

Resultados: Los resultados indican que las HMM son efectivas en la prevención de enfermedades cardíacas, pero su eficacia depende de la calidad de los datos. Estos resultados son prometedores, pero no universales en su aplicabilidad.

Conclusiones: En resumen, este estudio destaca la utilidad de las HMM en la detección temprana de infartos y su enfoque estadístico en medicina. Se enfatiza que no son infalibles y deben complementarse con otras opciones clínicas y métodos de evaluación en situaciones reales.

Originalidad: Este trabajo destaca por su enfoque estadístico y probabilístico en la aplicación de modelos de Markov ocultos (HMM) en el análisis médico, aportando una perspectiva innovadora y facilitando la comprensión de su utilidad en el campo de la medicina.

Limitaciones: Se reconoce que existen limitaciones, como la dependencia de la calidad de los datos y la aplicabilidad variable en casos clínicos. Estas limitaciones deben considerarse en el contexto de su implementación en la práctica médica

Palabras clave: Procesos estocásticos, cadenas de Markov, cadenas de Markov ocultas, ECG.

Resumo

Introdução: Este artigo, desenvolvido entre 2022 e 2023 no âmbito dos Processos Estocásticos Aplicados pelo grupo SciBas da Universidade Distrital Francisco José de Caldas, foca-se no papel das cadeias de Markov ocultas (HMM) na previsão de doenças cardiovasculares.

Problema: O problema abordado é a necessidade de melhorar a detecção precoce de doenças cardíacas, e destaca como as HMM podem lidar com a incerteza nos dados clínicos e detectar padrões complexos.

Objetivo: Avaliar o uso de modelos de Markov ocultos (HMM) na análise de eletrocardiogramas (ECG) para a detecção precoce de doenças cardiovasculares.

Metodologia: A metodologia inclui uma revisão da literatura sobre a relação entre as HMM e as doenças cardiovasculares, seguida da aplicação de HMM para prevenir infartos e abordar a incerteza nos dados clínicos.

Resultados: Os resultados indicam que as HMM são eficazes na prevenção de doenças cardíacas, mas sua eficácia depende da qualidade dos dados. Esses resultados são promissores, mas não universais em sua aplicabilidade.

Conclusões: Em resumo, este estudo destaca a utilidade das HMM na detecção precoce de infartos e seu enfoque estatístico na medicina. Enfatiza-se que não são infalíveis e devem ser complementadas com outras opções clínicas e métodos de avaliação em situações reais.

Originalidade: Este trabalho se destaca pelo enfoque estatístico e probabilístico na aplicação de modelos de Markov ocultos (HMM) na análise médica, proporcionando uma perspectiva inovadora e facilitando a compreensão de sua utilidade no campo da medicina.

Limitações: Reconhece-se que existem limitações, como a dependência da qualidade dos dados e a aplicabilidade variável em casos clínicos. Essas limitações devem ser consideradas no contexto de sua implementação na prática médica.

Palavras-chave: Processos estocásticos, cadeias de Markov, cadeias de Markov ocultas, ECG.

1. INTRODUCTION

In the 1970s and 1980s, the study of hidden Markov models (HMMs) was initiated, introducing the concept of short-term memory [1]. HMMs are probabilistic models that describe observable events that depend on unobservable internal factors [2], based on the Markov property that the probability of transitioning to a future state depends only on the current state and not on previous states.

Their applications have been diverse, encompassing facial recognition, speech recognition, genetic prediction, bioinformatics analysis, automatic classification of electrocardiograms (ECGs), neuronal activity in the visual cortex, epileptic seizures, tracking the movement of living organisms using ultrasound, aligning protein structure sequences, detecting homogeneous segments in DNA sequences, learning non-singular phylogenies, genetic mutation analysis of HIV sequences, and human genome analysis [3]–[5].

However, it is in the clinical field where the application of HMMs has had a significant impact, facilitating the early detection of cardiovascular diseases. This application is relevant given that these diseases are one of the leading causes of death worldwide, including Colombia. During the fourth quarter of 2022 and the months that have passed in 2023, an average of 44,942 deaths attributed to acute myocardial infarction (AMI) were reported in the country [6].

In this perspective, clinical research has decided to transition to the use of HMMs in order to employ large volumes of data to enrich and identify patterns from their analysis. An example of this is based on the use of electrocardiogram (ECG) signals, which, although they are examinations in which large volumes of information are obtained, only a fraction of it is analyzed, ignoring other information that may be of clinical interest and with which patterns can be obtained to predict cardiovascular events [7].

Specifically, an ECG is the graphical interpretation of the electrical activity recorded on the skin of the electric field that originates in the heart [8]; it provides information in the diagnosis of heart diseases because it is non-invasive, simple, and accessible. Consequently, the ECG is the subject of ongoing research to reduce subjectivity and the time spent interpreting voluminous records [9]–[11]. As it is an ideally periodic signal, it allows marking an elemental beat; the heart rate, then, can be estimated by detecting the QRS complex of an ECG signal and the time interval between successive QRS complexes (also known as the R-R interval) is used to detect premature ectopic beats [8].

This paper proposes an explanation of the use of the HMM (Hidden Markov Model) with ECG (Electrocardiogram) data, introducing the concepts and examples applied in [12] to clearly understand the construction of these models. For this purpose, hypothetical data related to the hidden and observable states of an ECG are used to study the health status of a patient and determine the probability of three states: 1) absence of myocardial infarction, 2) arrhythmia and 3) presence of myocardial infarction. In this regard, three key characteristics of each beat will be analyzed: QRS complex duration, ST-segment elevation and T-wave morphology. Initially, these states will form the Markov chain model and will be analyzed and interpreted taking into account their properties. Finally, we will try to answer key questions about the probability of the last signal emitted from a specific state, the probability of transition to a specific state and the probability of emission of a specific signal after a sequence of observed signals.

1.1 Research Background

In recent years, the use of Hidden Markov Models (HMMs) with electrocardiogram (ECG) data has enabled predictions for assessing the risk of certain diseases, particularly cardiovascular diseases. Remarkably, the significance of such signal analysis in clinical research has grown to such an extent that, even before the 2000s, various research centers began constructing public databases based on electrocardiographic signals to facilitate the development of predictive models for cardiovascular diseases by future researchers [13]–[17]. These databases, along with clinical data, have allowed for the detection of premature ventricular contractions (PVC) to predict cardiac arrhythmias in patients, using normal ECGs as a reference. Similarly, Nolle et al. (1987) [7] designed a computerized system based on HMM for real-time arrhythmia monitoring in hospitalized patients, achieving control over potential events [18]. Other research, such as that by Cheng and Chan [19] focused on designing models for classifying ECG signals into normal beats, premature ventricular contractions, and fusion of ventricular and normal beats, achieving classification accuracies of over 50%. In the same vein, research studies have been conducted, such as the one led by Vanderlei Filho et al. in 1999 [20].

In recent years, studies aiming to identify arrhythmias, based on analyses that include HMM, have become more prevalent [21]–[25]. This is because they have achieved high levels of accuracy in detecting the occurrence of these events, thus presenting alternative avenues for clinical research focused on preventing serious illnesses.

In 2015, Pimentel et al. [26] designed a multimodal physiological model using a hidden semi-Markov model. They integrated various heartbeat signals from electrocardiograms (ECG), arterial blood pressure (ABP), and photoplethysmograms (PPG) to predict whether alarms related to abnormal heartbeats in intensive care unit (ICU) patients were not false positives resulting from cardiac disturbances. This approach addressed the issue of misinterpretation when analyzing each of these signals separately, and they concluded that their model accomplished this task with precision.

Furthermore, a study was conducted using discrete-state Markov models and random forests for the early detection of atrial fibrillation (arrhythmias) based on ECG signals. The aim was to achieve higher precision and certainty in identifying these events, achieving sensitivity and specificity levels exceeding 90%. This allowed for rapid event identification, reducing high mortality rates and health deterioration [21]. Similarly, an investigation starting from a public database (MIT-BIH Arrhythmia) employed HMM to classify arrhythmias based on ECG data, resulting in a model with an overall accuracy and sensitivity of 99.7% [27]. In 2022, Kwok et al. [28], utilized a hidden semi-Markov model, wavelet analysis, and random forest to classify ECG rhythms during cardiopulmonary resuscitation (CPR). This provided new tools for first responders in emergencies to have a better understanding of the patient's condition.

From ECG signals, various other models based on HMM have been designed. The study conducted by Levin and colleagues in 2015 aimed to detect silent atrial fibrillation in patients with recent cerebrovascular accidents, with the goal of improving patients' quality of life and predicting potential cardiovascular outcomes [29]. A study led by Tison and colleagues in 2019 [30], employed ECG signals to achieve two objectives: 1. Estimate structural measures and cardiac function, and 2. Detect and track cardiovascular diseases such as cardiac amyloidosis, hypertrophic cardiomyopathy, and pulmonary arterial hypertension. They concluded that there is an expansion of clinical insights that can be derived from this type of examination when applying computerized interpretation algorithms.

In a similar vein, a study conducted in 2018 [31], found that by estimating atrial repolarization from ECG signals, it is possible to diagnose cardiovascular diseases more easily. This phenomenon serves as a crucial marker and an early sign of lower-level cardiovascular conditions, such as arrhythmias or acute atrial infarctions. Additionally, episodes of apnea-bradycardia have been observed in various studies [32], [33], including one using Coupled Hidden Semi-Markov Models (CHSMM) [34] and another employing layered HMMs, both based on electrocardiographic data. This detection contributes to the rapid diagnosis of this phenomenon in premature infants, a population where it occurs most frequently, thus avoiding neurological developmental issues and mortality [32], [33], [35].

Studies that are more complex propose the use of neural networks in conjunction with HMM for the prediction of potential cardiovascular events, such as the presence of arrhythmias [25], [36]–[43].

Additional research includes stress classification, particularly affective stress, using ECG signals with HMM models, contributing to the emerging field of affective computing, which focuses on recognizing and processing human emotions for therapeutic purposes [44]. Others are dedicated to the automatic staging of sleep using ECG, aiming to identify specific types of sleep disorders in patients. The designed model successfully identified different sleep stages (deep sleep, REM sleep, and wakefulness) with accuracies ranging from 76% to 85%. This leads to the conclusion that these methods can serve as an alternative to sleep staging, differing from examinations such as polysomnography, which is cost-intensive [45].

The article is structured as follows: initially, a background and notation of Markov chains and HMMs is established; then, the methodology for HMMs is established as a study model and prevention mechanism for anomalous cardiac events; subsequently, they are exemplified through hypothetical data in a timely manner; finally, the discussion of these results is carried out and the conclusions are established.

2. BACKGROUND AND NOTATION

In the following, the data obtained from ECGs are understood as in [12], defining the heart as a muscular organ whose function is to pump blood throughout the human body at an average rate of 50-100 beats per minute by contractions (systole) and relaxations (diastole) of the myocardium. The ability of heart cells to generate and conduct electrical impulses in each cardiac contraction translates the electrical activity of the heart into a tracing of lines, peaks, and valleys that configure waves [30], [46].

To incorporate the analysis with HMMs, the waves are noted as follows: QRS records the depolarization of the ventricles; ST represents the distance between the QRS complex and the T wave and, together with the ST, records the ventricular repolarization.

The concepts and definitions related to Markov chains are taken from [47].

A stochastic process is defined as a collection of random variables X_n, where n is an index that represents time. That is, each X_n is a random variable that represents the state of the process at time n. The evolution of the process over time is determined by the transition probabilities from one random variable to another. These transition probabilities can depend on the current state of the process and time. In the case of considering an ECG signal as a stochastic process $X_n \leq 0$, X is the random variable that records the following characteristics: duration of the QRS complex, ST segment height, and T wave, and classifies them as normal (0) or abnormal (1). The acquisition of the data is considered periodically, so the stochastic process is a discrete-time series at equidistant instants [27], [48], seen as a collection of random variables X_n parameterized by a set T or parametric space where the variables take values in a set S or state space. As the duration of the QRS complexes, the ST waves, and the T waves, present a simplified temporal dependency, the Markov chain approach is used for their analysis.

In the case at hand, the Markov chain is composed of a finite or countable set of states and a transition matrix that specifies the transition probabilities between the states. The transition matrix indicates the probability of transitioning from one state to another in a single time step. For any integer n≠0, and for any state, $x_0,\cdot,x_{n+1}$, we have: $p(x_{n+1}|x_0,\cdot,x_{n+1})=p(x_{n+1})|x_n)$. If we consider *n+1* as future time, n as present time, and the times 0,1,⋯,*n*-1 as past time, we can calculate the joint distribution of the variables $x_0,x_1,\cdot,x_n$ as follows:

$$p(x_0, x_1, \cdots, x_n) = p(x_0)p(x_1|x_0)\cdots p(x_n|x_{n-1})$$

The Markov chain is considered homogeneous when the transition probabilities P_{ij} are independent of n - the number of steps or beats: the probability of transitioning from state i to state j remains constant over time; if a patient is in the state “presence of infarction”, the probability of transitioning to the arrhythmia state or to the absence of infarction state in the next step will be constant, regardless of when the transition occurs.

On the other hand, in order for the transition matrix to be considered stochastic, it must meet the following criteria:

(1) $$P_{ij} \geq 0 \;\; y \;\; \sum P_{ij} = 1$$

On the other hand, the Chapman-Kolmogorov equation states that for any pair of integers *r* and *n* that 0 ≤ r ≤ n, and for any two states *I* and *j* it holds that:

(2) $$p_{ij}(n) = \sum_k p_{ik}(r)p_{kn}(n-r)$$

In this sense, the following notation is used for the states:

1. Absence of arrhythmia
2. Arrhythmia
3. Presence of infarction

It is important to note that the probabilities for transitioning from one state to another consider the evaluation of all possible paths to reach State 3, starting from State 1. Therefore, the inequality

(3) $$P_{ij}(n) \geq P_{ik}(r)P_{kj}(n-r)$$

The transition probability in n steps $P_{ij}(n)$ is given by (i,j) entry of n-nith power of the transition matrix $P_{ij}(n) = (P^n)_{ij}$.

The matrix A is diagonalizable, or can be written $A = QDQ^{-1}$, where D is a diagonal matrix; thus

(4) $$A^n = QDQ^{-1}.$$

Each element (i,j) of the matrix A^n corresponds to the probability of transitioning from state $P_{ij}(n)$.

On the other hand, communication between the states of the Markov chain is defined as the accessibility relation between state i and j, represented by $i \rightarrow j$, if there exists an integer $n \geq 0$ such that $P_{ij}(n) > 0$. This relation is an equivalence relation (reflexive, symmetric, and transitive). Two states, i and j, are considered communicating represented by $i \leftrightarrow j$ and $i \rightarrow j$ and $j \rightarrow i$. The communication class of state i is denoted by $C(i)$. Therefore, $i \leftrightarrow j$ if only if $C(i) = C(j)$.

Now, the period f of the state in the Markov chain is a non – negative integer that is calculated for each state and is related to the long – term behavior of the chain. The calculation of the period f at state i is as follows:

(5) $$d(i) = m.c.d\{n \leq 0 : p_{ii}(n) \leq 0\}$$

The study of the first time a Markov chain visits a particular state is carried out using an equation that relates it to the transition states. In this case, for each $n \geq 1$ the number $f_{ij}(n)$ denotes the probability that a chain that starts in state i, reaches state j for the first time in exactly n steps, that is:

(6) $$f_{ij}(n) = P(X_n = j, X_{n-1} \neq j, \cdot X_1 \neq j | X_0 = i)$$

or $f_{ij}(n) = P(\pi_{ij} = n)$, that is: the probability that τ_{ij} takes the value n, or that the first visit to state j starting from state i occurs in n steps. As a consequence, we have that for each:

(7) $$n \geq 1, p_{ij}(n) = \sum_{k=1}^{n} f_{ij}(k) p_{jj}(n-k).$$

On the other hand, a state *i* is considered recurrent if the probability of returning to *i* eventually, starting from *i,* is equal to 1. In the same way, a non-recurrent state is called transient, which means that the previous probability is less than 1. A state *i* is recurrent if $f_{ii} = 1$; while it is transient when $f_{ii} < 1$. Every finite chain contains at least one recurrent state, which implies that there are no finite transient chains.

A non – empty collection of states ϒ is closed if no state outside of ϒ is accessible from any state within ϒ; that is, for any $i \in ϒ$ y $j \in ϒ$, $i \nrightarrow j$. This means that, given that the chain is irreducible with a single class, it can be concluded that it is closed.

The number of visits is defined as: $N_{ij}(n) = \sum_{k=1}^{n} 1_{X_k=j}$ when $X_0 = i$. This property indicates the number of visits that the chain makes to state *j* starting from *i* in *n* steps. When the chain is irreducible, recurrent, and closed, it is the case that, if the chain starts in a state *i* with probability 1, it visits this state an infinite number of times in an infinite amount of time, as indicated by the following expressions:

(8) $$\mathrm{P}(N_{ij} = \infty) = \begin{cases} 0, & \text{if } j \text{ is transient} \\ f_{ij}, & \text{if } j \text{ is recurrent} \end{cases}$$

(9) $$P(N_{ij} < \infty) = \begin{cases} 1, & \text{if } j \text{ is transient} \\ 1 - f_{ij}, & \text{if } j \text{ is recurrent} \end{cases}$$

The following proposition is held.

(10) $$\sum_{n=1}^{\infty} p_{ii}(n) = \infty \text{ if } j \text{ is recurrent}$$

(11) $$\sum_{n=1}^{\infty} p_{ii}(n) = \ \&\text{if } j \text{ is transient}$$

The mean recurrence time of a recurrent state *j*, from state *i*, is defined along with the expectation of τ_{ij}, and is denoted by μ_{ij}; $\mu_{ij} = E(\tau_{ij}) = \sum_{n=1}^{\infty} n f_{ij}(n).$

A recurrent state *i* is said to be: a) Positive recurrent if $\mu_i < \infty$; b) Null recurrent if $\mu_i = \infty$. There are no null recurrent states in finite Markov chains in a finite and irreducible chain; all its states are positive recurrent, hence $\mu_i < \infty$ for all the three states. Therefore, an irreducible recurrent chain is positive.

On the other hand, a probability distribution $\pi_0 = (\pi_0, \pi_1, \cdots)$ is stationary or invariant for a Markov chain with transition probability matrix $P = (p_{ij})$, if

(12) $$\pi_j = \sum_i \pi_i p_{ij}.$$

That is, for any natural number n, it holds that $\pi = \pi P^n$; which indicates that π is also a stationary distribution for the matrix P^n given in the recurrence and transience property. Every Markov chain that is irreducible and positive recurrent has a unique stationary distribution given by

(13) $$\pi_j = \frac{1}{\mu_j}.$$

If the chain is also aperiodic, it holds that:

(14) $$lim\; p_{ij}(n) = \pi_j$$

Finally, chains are considered regular when there exists a natural number n in their transition probability matrix such that $p_{ij}(n) > 0$, for any states i and j. And a chain is reversible in time if it is irreducible with transition probabilities P_{ij} and stationary distribution π, for any states i y j; that is:

(15) $$\pi_i p_{ij} = \pi_j p_{ji}$$

Furthermore, with respect to hidden Markov models (HMMs); they are established from a sequence of a random variables $Y=\{Y_n\}_{n\geq 0}$ that are assumed to occur after the realization of a Markov chain $X=\{X_n\}_{n\geq 1}$ that is not observed [49].

In this way, HMM components are: $S = \{s_1, s_2, \ldots, s_N\}$, hidden components:

- $X = \{x_1, x_2, \ldots, x_M\}$, observable states.
- $X = \{a_{ij}\}, a_{ij} = P(s_n = j | s_{n-1} = i)$, probability session

- $B=\{b_i(t)\}, b_i(t) = P(x_n = o_t \mid s_n = i)$, the probability of emitting an observable state from o_t state *i*
- $\pi=\{\pi_i\}$, $\pi_i = P(s_0 = 1)$, probability that stars in the ninth hidden state.

The probability that the first observation is emitted by state *i* is denoted as

(16) $$F_1(i) = \pi_i P(1|i).\ i$$

The probability that the *n*-th observation is emitted by state j, is denoted as:

(17) $$F_n(j) = P(S_n|j) \sum_i F_{n-1}(i) P_{ij}$$

The probability that the last observation is emitted by a particular state is denoted as:

(18) $$P(X_n = j|S_n = n) = F_n(j) / \sum_i F_n(i).$$

The probability that, once a set of observations has been emitted which ended in a particular state, a particular observation occurs is:

(19) $$\mathrm{P(X_n = j|S_{n-1}) = \sum_i P(X_{n-1} = i)\ P_{ij}}.$$

The probability that, once, a set of hidden states has been emitted, a particular hidden state occurs is:

(20) $$P(S_{n+1} = a|S_n) = \sum_i P(S_{n+1} = a|X_n = i, S_n) P(X_{n+1} = i|S_n)$$

And the probability that a set of hidden states occurs is denoted as:

(21) $$P(S^n = s_n) = \sum_i F_n(i)$$

3. METHODOLOGY:

Incorporating Markov chains provides a framework for understanding and predicting the probabilistic evolution of systems that change over time.

In this perspective, statistical models assume that a time series can be described by a parametric random process, whose parameters can be estimated in a well-defined way. HMMs belong to this category of models [50], [51].

Initially, a Markov chain is constructed, as shown in Figure 1.

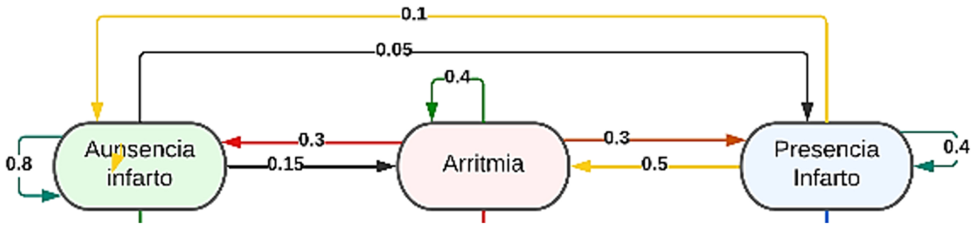

Figure 1. Markov chain for the three states: Absence of Myocardial Infarction, Arrhythmia, and Presence of Myocardial Infarction.
Source: own elaboration.

The incorporated data comes from the analysis given by expert judgment of the SciBas research group. This allows for graphical visualization of different properties that start from the transition matrix, which is schematized in Table 1, where the probabilities of transitioning between the three states are shown.

Table 1. Transition Matrix for the Markov chain.

	Absence of MI	Arrhythmia	Presence of MI
MI Absence	0.8	0.15	0.05
Arrhythmia	0.3	0.4	0.3
Presence of MI	0.1	0.5	0.4

Source: own elaboration.

Assuming the availability of a sample of ECG signals that capture the electrical information of the heart during several beats; an HMM is now used to predict the possibility of a myocardial infarction by analyzing three key characteristics of each beat: duration of the QRS complex, height of the ST segment and morphology of the T wave, providing relevant information about heart health.

In this way, the hidden Markov chain expresses the properties in the clarification of possible recurrences of symptoms or warning signs; and it also supports learning to accurately identify the initial moment in which these signs begin to manifest themselves. This approach provides a comprehensive and nuanced view of the evolution of the patient's health status, paving the way for earlier detection and a more informed understanding of the probability of occurrence of a myocardial infarction. The hidden Markov chain is shown in Figure 2.

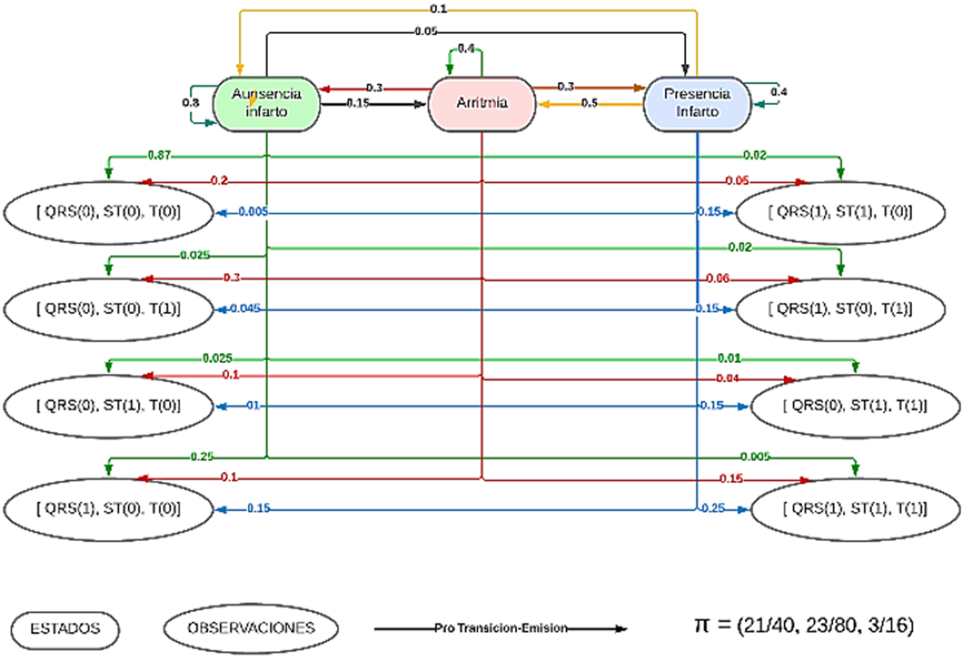

Figure 2. Hidden Markov chain (headed by the Markov chain in Figure 1).
Source: own elaboration.

Subsequently, an example is presented to understand how HMMs are applied to the detection of acute myocardial infarction. Given an ECG signal with 2 beats; an HMM is applied by dividing the signal into 2 intervals corresponding to the duration of a beat, and the mentioned characteristics are extracted. Each feature is classified as normal (0) or abnormal (1), generating 8 possible combinations of values. These combinations represent the observations for the analysis: assuming that the HMM classifies the first beat as absence of infarction and the second beat as arrhythmia. This suggests a change in the patient's health status between both beats, see Table 2. Table 2 shows the emission matrix that shows the probabilities for 8 observations.

Table 2. Emission Matrix with probabilities of 8 observations.

	[QRS(0), ST(0), T(0)]	[QRS(0),ST(0), T(1)]	[QRS(0),ST(1),T(0)]	[QRS(1),ST(0),T(0)]	[QRS(1), ST(1), T(0)]	[QRS(1), ST(0), T(1)]	[QRS(0), ST(1), T(1)]	[QRS(1), ST(1), T(1)]
MI Absence	0.87	0.025	0.025	0.025	0.02	0.02	0.01	0.005
Arrhythmia	0.2	0.3	0.1	0.1	0.05	0.06	0.04	0.15
Presence of MI	0.005	0.045	0.1	0.15	0.15	0.15	0.15	0.25

Source: own elaboration.

Subsequently, 4 fundamental questions are formulated for the detection of infarctions, and the importance of each inquiry:

1. What is the probability that the last observed signal is emitted by a particular state? Calculating this probability is essential to identify the current state of the system and its relationship to the risk of infarction. This provides information about the patient's current condition and their possible prognosis. It is important to highlight that this information is valuable for the attending physician, as it allows them to anticipate the event and predict possible solutions to an MI.
2. What is the probability that once a set of observed signals is emitted, a particular state is transitioned to? Calculating this probability is crucial for medical staff as it allows them to understand how the evolution of states relates to the progression of the risk of infarction. This information helps to evaluate the change in the patient's health over time.
3. What is the probability that once a set of observed signals is emitted, a particular signal is emitted? Calculating this probability is essential to understand how past signals are related to the possibility of a signal associated with an imminent infarction. This allows for early detection of adverse cardiac events and preparation for future events.
4. What is the probability of the set of observed signals given? Calculating this probability is fundamental to determine how well the model fits the real data and, therefore, how reliable the HMM is in predicting infarctions. This provides a measure of the model's accuracy in clinical practice.

Finally, a comparative table is presented with two sets of observed signals.

4. RESULTS:

Initially, it is essential to understand how the system under study behaves and how the states evolve over the cardiovascular event. To this end, the results are distinguished as follows:

4.1. Properties of Markov chains:

Homogeneity implies that the probability of transitioning from state i to state j remains constant over time.

The stochastic matrix property (1) is verified in Table 1.

For the case of Equation (2), a particular case is proposed:

$$\begin{aligned} P_{12}(3) &= (P_{11}P_{11}P_{12}) + (P_{11}P_{12}P_{22}) + (P_{11}P_{13}P_{32}) + (P_{12}P_{21}P_{12}) + (P_{12}P_{22}P_{22}) \\ &\quad + (P_{12}P_{23}P_{32}) + (P_{13}P_{31}P_{12}) + (P_{13}P_{32}P_{22}) + (P_{13}P_{33}P_{32}) \\ &= (0.8 \cdot 0.8 \cdot 0.15) + (0.8 \cdot 0.15 \cdot 0.4) + (0.8 \cdot 0.05 \cdot 0.5) + (0.15 \cdot 0.3 \cdot 0.15) \\ &\quad + (0.15 \cdot 0.4 \cdot 0.4) + (0.15 \cdot 0.3 \cdot 0.5) + (0.05 \cdot 0.1 \cdot 0.15) \\ &\quad + (0.05 \cdot 0.5 \cdot 0.4) + (0.05 \cdot 0.4 \cdot 0.5) \\ &= 0.238 \qquad 0.4 \end{aligned}$$

The consideration of the steps to reach State 3 (infarction) from State 2 (arrhythmia), is observed in:

$$P_{32}(2) = P_{31}P_{12} + P_{32}P_{22} + P_{33}P_{32} = (0.1 \cdot 0.4) + (0.5 \cdot 0.4) + (0.4 \cdot 0.5) = 0.44$$

The inequality (3) can be represented, in the specific case $P_{13}(1)\,P_{32}(2)$, which results in: $P_{13}(1)\,P_{32}(2) = 0.05 \cdot 0.44 = 0.022$. Besides, it is important to mention that P_{ij} (n) can be represented by P_{12}, with a value of 0.238. Thus, inequality (3) is confirmed.

This inequality indicates that it is more likely to transition from a state of absence of infarction to arrhythmia than to first transition to infarction and then to a state of arrhythmia. In other words, it is more likely to experience the symptoms of a possible infarction, such as an arrhythmia, before simply presenting an infarction at any time and then, presenting the symptoms of an arrhythmia.

Then, the diagonalization (4) is given by $A = \begin{pmatrix} 0.8 & 0.15 & 0.05 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.5 & 0.4 \end{pmatrix}$

$$Q = \begin{pmatrix} 1 & -\frac{11}{17} & \frac{1}{11} \\ 1 & \frac{9}{17} & -\frac{9}{11} \\ 1 & 1 & 1 \end{pmatrix} D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 \end{pmatrix} Q^{-1} = \begin{pmatrix} \frac{21}{40} & \frac{23}{80} & \frac{3}{16} \\ -\frac{17}{24} & \frac{17}{48} & \frac{17}{48} \\ \frac{11}{60} & -\frac{77}{120} & \frac{11}{24} \end{pmatrix}$$

$$A^n = \begin{pmatrix} \frac{11 \cdot 3^{n-1} \cdot 5^{1-n} + 21}{40} & \frac{23 - 11 \cdot 3^{n-1} \cdot 5^{1-n}}{80} & \frac{3 \cdot 5^n - 11 \cdot 3^{n-1}}{16 \cdot 5^n} \\ \frac{21 - 3^{n+1} \cdot 5^{1-n}}{40} & \frac{3^{n+1} \cdot 5^{1-n} + 23}{80} & \frac{3^{n+1} + 3 \cdot 5^n}{16 \cdot 5^n} \\ \frac{21 - 17 \cdot 3^{n-1} \cdot 5^{1-n}}{40} & \frac{17 \cdot 3^{n-1} \cdot 5^{1-n} + 23}{80} & \frac{17 \cdot 3^{n-1} + 3 \cdot 5^n}{16 \cdot 5^n} \end{pmatrix}$$

For the presented Markov chain, C(1) = 1, 2, 3, which indicates that the chain is irreducible. This implies that all states communicate with each other in a finite number of steps.

To find the period, we have Figure 3.:

$1 \to 1$	· $1 \to 1{,}1 \to 1$	· $1 \to 1, 1 \to 1,\ 1 \to 1$
	· $1 \to 2{,}2 \to 1$	· $1 \to 1, 1 \to 2,\ 2 \to 1$
	· $1 \to 3{,}3 \to 1$	· $1 \to 1, 1 \to 1, 3 \to 1$
		· $1 \to 2, 2 \to 2,\ 1 \to 1$
		· $1 \to 2, 2 \to 3,\ 2 \to 1$
		· $1 \to 2, 2 \to 3,\ 3 \to 1$
		· $1 \to 3, 3 \to 3,\ 1 \to 1$
		· $1 \to 3, 3 \to 1,\ 2 \to 1$
		· $1 \to 3, 3 \to 1,\ 3 \to 1$

Figure 3. Calculation of the period of Markov chain 1.
Source: own elaboration

It can be observed that for (5), d(1) = m.c.d {1,2,3} = 1

Considering that the period is a class property, the chain is irreducible; it follows that the period of the 3 states is 1. This implies that, if the chain is in the arrhythmia state, it only requires one instant of time (beat) to return to that state, and this applies to each of the three states.

The Equation (6) can be exemplified as follows:

$$
\begin{aligned}
&f_{12}(1) = 0.15\\
&f_{12}(2) = (p_{11}p_{12}) + (p_{13}p_{32})\\
&= (0.8 \cdot 0.15) + (0.05 \cdot 0.5)\\
&= 0.145\\
&f_{12}(3) = (p_{11}p_{11}p_{12}) + (p_{11}p_{13}p_{32}) + (p_{13}p_{31}p_{12}) + (p_{13}p_{33}p_{32})\\
&= (0.8 \cdot 0.8 \cdot 0.15) + (0.8 \cdot 0.05 \cdot 0.15) + (0.05 \cdot 0.1 \cdot 0.15) + (0.05 \cdot 0.4 \cdot 0.5)\\
&= \ 0.12675\\
&p_{22}(2) = (p_{22}p_{22}) + (p_{21}p_{12}) + (p_{23}p_{32})\\
&= (0.4 \cdot 0.4) + (0.3 \cdot 0.15) + (0.3 \cdot 0.5)\\
&= 0.355
\end{aligned}
$$

Therefore, it follows that for Equation (7):

$$
\begin{aligned}
p_{12}(3) &= \sum_{k=1}^{3} f_{12}(k)p_{22}(n-3)\\
&= f_{12}(1)p_{22}(2) + f_{12}(2)p_{22}(1) + f_{12}(3)p_{22}(0)\\
&= (0.15 \cdot 0.355) + (0.145 \cdot 0.4) + (0.12675 \cdot 1)\\
&= 0.238
\end{aligned}
$$

Therefore, the probability of presenting an arrhythmia for the first time while in the absence of infarction state is 23.8%

Specifically, recurrence implies that, if we start from a state of absence of arrhythmia, we will return to that state with a probability of 1. On the other hand, a non-recurring state is called transient, which means that the previous probability is less than 1. This property can be exemplified using the probability of transition in n steps (4).

In the specific case of State 3 (presence of infarction), it follows that for Equation (10):

$$\sum_{n=1}^{\infty} p_{33}(n) = \sum_{n=1}^{\infty} \frac{11 * 3^{n-1} * 5^{1-n} + 21}{40} = \infty$$

Therefore, it is concluded that State 3 is recurrent.

Since the Markov chain has at least one recurrent state and, being irreducible, it can be concluded that the chain as a whole is recurrent. This means that each state of the chain can be visited an infinite number of times in an infinite amount of time. In this case, the property suggests that once a patient presents arrhythmia or infarction, they will always be prone to experiencing those pathological events again. However, in terms of infinite time, it is possible for a person to experience a pathological state (arrhythmia or infarction) only once in their lifetime.

Since in the chain $\mu_i < \infty$ for each of the three states, it can be concluded that this will be the number of beats it will take a patient to present either of the two pathological states. Follow from Equation (12):

$$\mu_1 = \frac{40}{21} = 1.9 \quad \mu_2 = \frac{80}{23} = 3.47 \quad \mu_3 = \frac{16}{3} = 5.3$$

The above implies that the return to the pathological States 2 and 3, arrhythmia and presence of infarction, requires more time or number of beats to occur. This means that the patient is less likely to remain in the 'absence of infarction' state.

Since the chain is irreducible, recurrent, and closed, if the chain starts in state i with probability 1, it visits this state an infinite number of times in an infinite amount of time. This means that a patient who has experienced an arrhythmia or infarction event is likely to continue to experience episodes of changes in their ECG signal recordings for the rest of their life.

Given that the chain is recurrent, the number of visits to any state is infinite; this can be seen as a person living their entire life with various episodes of pathological states, for example, a person with arrhythmias will constantly move from a state of absence of infarction to one of arrhythmia. At the same time, Equation (9) is interpreted as the probability that a person will present a limited number of pathological states; that is, that they have presented a state of health of arrhythmia or presence of infarction once in their life, or that it occurs under very specific conditions, such as severe scares or infrequent events such as jumping out of an aircraft with a parachute or being exposed to an intense emotional situation.

Based on the properties reviewed, the irreducible chain is positive recurrent. This means that it takes a finite amount of time for the chain to return to a recurrent state i if it starts from it; which can be interpreted as, "a person who presents arrhythmia or presence of infarction with some frequency will present signs of those symptoms".

Since the chain is also aperiodic, it follows that for Equation (14), the π values will be:

$$\pi_1 = \lim_{n\to\infty} p_{11}(n) = \lim_{n\to\infty} \frac{11\cdot 3^{n-1}\cdot 5^{1-n}+21}{40} = \lim_{n\to\infty} \frac{11\cdot 3^{n-1}\cdot 5^{1-n}}{40} + \lim_{n\to\infty} \frac{21}{40}$$

$$= \frac{11}{40}\lim_{n\to\infty} 3^{n-1}\cdot 5^{1-n} + \frac{21}{40} = \frac{11}{40}\lim_{n\to\infty} \frac{3^n\cdot 5}{3\cdot 5^n} + \frac{21}{40} = \frac{55}{120}\lim_{n\to\infty}\left(\frac{3}{5}\right)^n + \frac{21}{40} = \frac{21}{40}$$

$$\pi_2 = \lim_{n\to\infty} p_{22}(n) = \lim_{n\to\infty} \frac{3^{n+1}\cdot 5^{1-n}+23}{80} = \frac{1}{80}\lim_{n\to\infty} 3^{n+1}\cdot 5^{1-n} \lim_{n\to\infty}\frac{23}{80}$$

$$= \frac{15}{80}\lim_{n\to\infty}\left(\frac{3}{5}\right)^n + \frac{23}{80} = \frac{23}{80}$$

$$\pi_3 = \lim_{n\to\infty} p_{33}(n) = \lim_{n\to\infty} \frac{17\cdot 3^{n-1}+3\cdot 5^n}{16\cdot 5^n} = \frac{17}{16}\lim_{n\to\infty}\frac{3^{n-1}}{5^n} + \lim_{n\to\infty}\frac{3}{16} = \frac{17}{48}\lim_{n\to\infty}\left(\frac{3}{5}\right)^n + \frac{3}{16}$$

$$= \frac{3}{16}$$

Then it follows that for Equation (13): $\pi = \left(\frac{21}{40}, \frac{23}{80}, \frac{3}{16}\right)$

Then, the distribution verified π is stationary (14).

$$\pi_1 = \frac{21}{40} = \frac{21}{40}0.8 + \frac{23}{80}0.3 + \frac{3}{16}0.1$$
$$\pi_2 = \frac{23}{80} = \frac{21}{40}0.15 + \frac{23}{80}0.4 + \frac{3}{16}0.5$$

It follows that: $\pi_1 = \frac{21}{40}$ $\pi_2 = \frac{23}{80}$ $p_{12} = 0.8$ $p_{21} = 0.3$

Then, as $\frac{21}{40}\cdot 0.8 = 0.42 \neq 0.08625 = \frac{23}{80}\cdot 0.3$; It follows that the chain is not reversible. This defines that it is not equally probable to transition from a state of infarction to a state of absence of infarction, as it is to transition from a state of absence of infarction to a state of infarction.

4.2 Properties of hidden Markov chains:

Equation (16) $F_1(i)$:

$$F_1(1) = \pi_1 p(1|1) = \frac{21}{40} * 0.87 \approx 0.4567$$

$$F_1(2) = \pi_2 p(1|2) = \frac{23}{80} * 0.2 = 0.0575$$

$$F_1(3) = \pi_1 p(1|3) = \frac{3}{16} * 0.05 \approx 0.0093$$

It is revealed that the probability of remaining in the state of “absence of infarction” on the next beat is significantly greater than the probability of transitioning from a pathological state (arrhythmia or presence of infarction) to a healthy state. In fact, the probability of emitting a healthy state is the lowest of all possibilities.

Equation (17) $F_2(j)$:

$$
\begin{aligned}
&F_2(1) = p(s_2|1)\sum_i F_1(i)p_{ij} \\
&= p(2|1)(F_1(1)p_{11} + F_1(2)p_{21} + F_1(3)p_{31}) \\
&= 0.025\big((0.4567 * 0.8) + (0.0575 * 0.3) + (0.0093 * 0.1)\big) \\
&\approx 0.00917 \\
&F_2(2) = p(s_2|2)\sum_i F_1(i)p_{ij} \\
&\qquad = p(2|2)(F_1(1)p_{12} + F_1(2)p_{22} + F_1(3)p_{32}) \\
&\qquad = 0.1\big((0.4567 * 0.15) + (0.0575 * 0.4) + (0.00093 * 0.5)\big) \\
&\qquad \approx 0.00919 \\
&F_2(3) = p(s_2|3)\sum_i F_2(i)p_{ij} \\
&\qquad = p(2|3)(F_1(1)p_{13} + F_1(2)p_{23} + F_1(3)p_{33}) \\
&\qquad = 0.1\big((0.4567 * 0.05) + (0.0575 * 0.3) + (0.00093 * 0.4)\big) \\
&\qquad \approx 0.00404
\end{aligned}
$$

These results imply that a signal [0,1,0] that shows an abnormal value in the ST segment height, can be emitted by both the “absence of infarction” and “arrhythmia” states. This similarity, in the short term, can be attributed to a momentary activity that could have temporarily altered the person’s heart rate. However, it is unlikely that the signal is emitted by the “presence of infarction” state, as this state has a much lower emission probability. In other words, it is unlikely that a person would experience such a drastic change in their heart rate in a single beat.

Knowing these values, it is possible to answer the fundamental questions raised based on the calculations of Equation (18):

$P(X_2 = 1|S_2)$; $P(X_2 = 2|S_2)$; $P(X_2 = 3|S_2)$; $P(X_3 = 1|S_2)$; $P(X_3 = 2|S_2)$; $P(X_3 = 3|S_2)$; $P(S_3 = 1|S_2)$; $P(S_3 = 2|S_2)$; y $P(S^2 = s_2)$.

$$
\begin{aligned}
&P(X_2 = 1|S_2) = \frac{F_2(1)}{\sum_i F_2(i)} = \frac{0.00917}{0.0022} \approx 0.4093 \\
&P(X_2 = 2|S_2) = \frac{F_2(2)}{\sum_i F_2(i)} = \frac{0.00919}{0.0022} \approx 0.4101 \\
&P(X_2 = 3|S_2) = \frac{F_2(3)}{\sum_i F_2(i)} = \frac{0.0040}{0.0022} \approx 0.1843
\end{aligned}
$$

The probability of the sequence of observations ending in the state of absence of infarction or arrhythmia is very similar, while ending in the state of presence of infarction is much lower. This is consistent with the fact that a heart attack is unlikely to occur without presenting symptoms, such as a prolonged arrhythmia.

Equation (19):

$$
\begin{aligned}
&P(X_3 = 1|S_2) = \sum_i P(X_2 = i|S_2)p_{i1} \\
&= P(X_2 = 1|S_2)p_{11} + P(X_2 = 2|S_2)p_{21} + P(X_2 = 3|S_2)p_{31} \\
&= (0.4093 * 0.8) + (0.4101 * 0.3) + (0.1843 * 0.1) \\
&\approx 0.4689 \\
&P(X_3 = 2|S_2) = \sum_i P(X_2 = i|S_2)p_{i2} \\
&= P(X_2 = 1|S_2)p_{12} + P(X_2 = 2|S_2)p_{22} + P(X_2 = 3|S_2)p_{32} \\
&= (0.4093 * 0.15) + (0.4101 * 0.4) + (0.1843 * 0.5) \\
&\approx 0.3175 \\
&P(X_3 = 3|S_2) = \sum_i P(X_2 = i|S_2)p_{i3} \\
&= P(X_2 = 1|S_2)p_{13} + P(X_2 = 2|S_2)p_{23} + P(X_2 = 3|S_2)p_{33} \\
&= (0.4093 * 0.05) + (0.4101 * 0.3) + (0.1843 * 0.4) \\
&\approx 0.2172
\end{aligned}
$$

The probability of the sequence of observations ending in a state of absence of infarction or arrhythmia is very similar, while the probability of ending in the state of presence of infarction is lower. This means that the occurrence of a heart attack without previous symptoms (such as a prolonged arrhythmia) is unlikely.

Equation (20):

$$
\begin{aligned}
P(S_3 = 1|S_2) &= P(S_3 = 1|X_2 = 1)P(X_3 = 1|S_2) + P(S_3 = 1|X_2 = 2)P(X_3 = 2|S_2) \\
&\quad + P(S_3 = 1|X_2 = 3)P(X_3 = 3|S_2) \\
&= (0.87 * 0.4689) + (0.3175 * 0.2) + (0.2172 * 0.005) \\
&\approx 0.4725 \\
P(S_3 = 2|S_2) &= P(S_3 = 2|X_2 = 1)P(X_3 = 1|S_2) + P(S_3 = 2|X_2 = 2)P(X_3 = 2|S_2) \\
&\quad + P(S_3 = 2|X_2 = 3)P(X_3 = 3|S_2) \\
&= (0.025 * 0.4689) + (0.1 * 0.3175) + (0.1 * 0.2172) \\
&\approx 0.0651
\end{aligned}
$$

It is more likely to stabilize the heart rate and move to a state of absence of infarction, as the probability of maintaining a pathological state: arrhythmia or presence of infarction, is lower.

The probability of the observed signal set being given is calculated as follows: Equation (21)

$$P(S^2 = s_2) = \sum_i F_2(i)$$
$$= F_2(1) + F_2(2) + F_2(3)$$
$$\approx 0.00917 + 0.00919 + 0.00404$$
$$\approx 0.0224$$

Regarding the comparison between the observed events, two signals are established: 1) Transition from a state, whose 3 characteristics are classified as normal to a state where the ST characteristic is classified as abnormal. 2) Transition from a state, whose 3 characteristics are classified as normal, to a state where all 3 characteristics are classified as abnormal.

The resulting transitions can be seen in Table 3. They are analyzed to identify patterns in the evolution of the ECG signal characteristics and to evaluate their relevance in the detection of infarctions. The number indicates the question to which the results refer.

Table 3. Comparisons of transitions between two sets of observed signals: ([0,0,0],[0,1,0]), and ([0,0,0],[1,1,1]).

Transitions	S=([0,0,0],[0,1,0])	S=([0,0,0],[1,1,1])
1	• $P(X_2 = 1 \mid S_2) \approx 0.4093$ • $P(X_2 = 2 \mid S_2) \approx 0.4101$ • $P(X_2 = 3 \mid S_2) \approx 0.1843$	• $P(X_2 = 1 \mid S_2) \approx 0.0741$ • $P(X_2 = 2 \mid S_2) \approx 0.5342$ • $P(X_2 = 3 \mid S_2) \approx 0.3916$
2	• $P(X_3 = 1 \mid S_2) \approx 0.4689$ • $P(X_3 = 2 \mid S_2) \approx 0.3175$ • $P(X_3 = 3 \mid S_2) \approx 0.2172$	• $P(X_3 = 1 \mid S_2) \approx 0.2587$ • $P(X_3 = 2 \mid S_2) \approx 0.4205$ • $P(X_3 = 3 \mid S_2) \approx 0.3218$
3	• $P(S_3 = 1 \mid S_2) \approx 0.4725$ • $P(S_3 = 2 \mid S_2) \approx 0.0651$	• $P(S_3 = 1 \mid S_2) \approx 0.3107$ • $P(S_3 = 2 \mid S_2) \approx 0.1448$
4	• $P(S^2 = S_2) \approx 0.0224$	• $P(S^2 = S_2) \approx 0.1166$

Source: Own elaboration

Table 3 highlights significant differences between the two sets of observations. In relation to the first question, it is observed that it is more likely that the first set of observations will end in State 1, while the second set tends to end in State 2. With respect to the second question, it is appreciated that the first set of observations shows a higher probability of transitioning to State 1, while for the second set, the transition to State 2 is more likely. As for the third question, it is noted that, of the two observations proposed, both sets have a higher probability that the third observation will be 1. This is present in both sets of observations. Finally, when analyzing the fourth question, it is concluded that the second set of observations has a higher probability of AMI compared to the first.

5. DISCUSSION AND CONCLUSIONS

This article presents a mathematical analysis of the properties of Hidden Markov Models (HMMs) in the prediction of acute myocardial infarction (AMI) through the analysis of electrocardiographic signals (ECG). The results, based on hypothetical data, suggest that HMMs constitute an effective tool for early detection of AMI.

The investigated properties include homogeneity, irreducibility, recurrence (Equations 8-11), periodicity (Equation 5), stationary probability (Equation 12), and reversibility (Equation 15). First and foremost, the property of homogeneity is highlighted, demonstrating that transitions between different states of cardiac health (absence of infarction, arrhythmia, and presence of infarction) remain constant over time. This suggests that patients who have experienced events such as arrhythmias or infarctions have a high probability of experiencing them again in the future, underscoring the need for constant monitoring and appropriate treatment for these patients.

Additionally, it is observed that the hidden Markov chain is not reversible (Equation 15), meaning that it is not equally probable to transition from a state of infarction presence to one of absence, or vice versa, as transition probabilities between states differ. On the other hand, it is identified that the chain is irreducible and recurrent (Equations 8-11), implying that over an infinite time, a patient who has experienced arrhythmias or infarctions will likely continue experiencing them throughout their life. However, the chain's period is 1 (Equation 5), signifying that the return to a recurrent state from any state occurs after just one step. This is crucial for understanding the temporal dynamics of cardiac events.

The analysis reveals that the calculations yield a lower probability of emitting an abnormal signal when the patient is in a healthy state compared to a pathological one (Equation 18). This difference may be attributed to the data used in the analysis, emphasizing the importance of using real data for a more precise interpretation.

Ultimately, a comparison of the probabilities of cardiac state occurrences is conducted, highlighting differences in transition probabilities between states of infarction absence, arrhythmia, and infarction presence (Table 3). This suggests that certain patterns in the evolution of ECG signals may be indicative of infarctions based on their probabilistic value.

This study is compared to the research by Álvarez and Henao (2006) [12], which also employed HMMs to predict AMI. Both studies share the use of HMMs to model the temporal evolution of ECG signals and conclude that they are effective in detecting AMI. However, they differ in focus (mathematical versus applied), the number of states used (fewer in this article), additional concepts (PPCA excluded in this article), and consideration of clinical factors (excluded in this article).

Despite its contributions, this study has limitations, particularly in regarding the use of hypothetical data and the lack of consideration of relevant clinical factors. It is suggested to validate the results with real clinical data, consider the patient's medical history, and utilize additional diagnostic techniques such as magnetic resonance imaging. Furthermore, the use of tools like MATLAB is proposed to conduct more extensive and precise simulations for predicting cardiovascular events, including AMI.

Based on the identified limitations in the study, future research is recommended to validate the results with real clinical data, consider relevant clinical factors such as the patient's medical history, and employ advanced data analysis tools like MATLAB to conduct more extensive and precise simulations.

This study supports the effectiveness of Hidden Markov Models in early detection of AMI through ECG signal analysis, with the potential to anticipate serious cardiovascular events and enhance medical care. Advancing in this direction can have a significant impact on patient health.

In conclusion, the research has demonstrated that Hidden Markov Models are an effective tool for the early detection of acute myocardial infarction through ECG signal analysis. These models offer the possibility of anticipating serious cardiovascular events and ultimately saving lives. To conclude, the importance of interdisciplinary research and the application of advanced data analysis methods in the field of healthcare is emphasized. It is hoped that this work serves as a starting point for future research and contributes to the advancement of detection and prevention of cardiovascular diseases.

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