Interpolation synthesis of the controllers for a multi-motor electric drive system containing an elastically linked element

Síntesis de interpolación de los controladores para un sistema de accionamiento eléctrico multimotor que contiene un elemento enlazado elásticamente

Síntese de Interpolação dos Controladores para um Sistema de Acionamento Elétrico Multi-Motor Contendo um Elemento Ligado Elasticamente

Dang Nguyen Phu[1]
Tuan Vu Duc[2]
Vijender Kumar Solanki[3]
Lam Sinh Cong[4]
Duc-Tan Tran[5]

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[1] Le Quy Don Technical University
Email: npdang@lqdtu.edu.vn
ORCID: https://orcid.org/0000-0002-9412-2839

[2] University of Transport Technology
Email: tuanvd@utt.edu.vn
ORCID: https://orcid.org/0000-0002-0960-667X

[3] Faculty of Computer Science & Engineering, CMR Institute of Technology, Hyderabad, TS, India
Email: spesinfo@yahoo.com

[4] Faculty of Electronics and Telecommunication VNU University of Engineering and Technology
Email: congls@vnu.edu.vn

[5] Faculty of Electrical and Electronic Engineering, Phenikaa University, Hanoi 12116, Vietnam
Email: tan.tranduc@phenikaa-uni.edu.vn
ORCID: https://orcid.org/0000-0002-7673-388X

Abstract

Introduction: The multi-motor electric drive systems that include elastic conveyors are one example of typical systems with distributed parameters described by complex equations. Because of the elastic and distributive nature of the system parameters, the transfer function describing them is often a complex expression, containing the inertial and transcendental components.

Problem: The elastic and distributive nature of system parameters makes the precise control of tension and speed synchronously much more complicated than the centralized parameter system.

Methodology: We propose a numerical solution for synthesizing the regulators based on the real interpolation method to reduce computational capacity and synthesis error while preserving the characteristic properties of objects with distributed parameters.

Conclusion: The effectiveness of the proposed algorithm is verified by an experimental model of the two-motor electric drive system containing an elastic conveyor. The simulation and experimental results indicate that the control system with the received regulators operates stably and meets the required quality criteria.

Originality: The research findings can be applied in the development of central control and monitoring systems for automatic production lines with multi-motor drive systems that include conveyors.

Keywords: Automatic control system, Regulator synthesis, Objects with distributed parameters, Elastic conveyor, Transfer function.

Resumen

Introducción: los sistemas de accionamiento eléctrico multimotor que incluyen transportadores elásticos son un ejemplo de sistemas típicos con parámetros distribuidos descritos por ecuaciones complejas. Debido a la naturaleza elástica y distributiva de los parámetros del sistema, la función de transferencia que los describe suele ser una expresión compleja que contiene los componentes inercial y trascendental.

Problema: la naturaleza elástica y distributiva de los parámetros del sistema hace que el control preciso de la tensión y la velocidad sincrónicamente sea mucho más complicado que el sistema de parámetros centralizados.

Metodología: se propone una solución numérica para sintetizar los reguladores basada en el método de interpolación real para reducir la capacidad computacional y el error de síntesis preservando las propiedades características de los objetos con parámetros distribuidos.

Conclusión: la eficacia del algoritmo propuesto se verifica mediante un modelo experimental del sistema de accionamiento eléctrico de dos motores que contiene un transportador elástico. Los resultados de simulación y experimentales indican que el sistema de control con los reguladores recibidos opera de manera estable y cumple con los criterios de calidad requeridos.

Originalidad: los resultados de la investigación se pueden aplicar en el desarrollo de sistemas centrales de control y monitoreo para líneas de producción automáticas con sistemas de accionamiento multimotor que incluyen transportadores.

Palabras clave: Sistema de control automático, Síntesis de reguladores, Objetos con parámetros distribuidos, Transportador elástico, Función de transferencia.

Resumo

Introdução: Os sistemas de acionamento elétrico multimotor que incluem transportadores elásticos são um exemplo de sistemas típicos com parâmetros distribuídos descritos por equações complexas. Devido à natureza elástica e distributiva dos parâmetros do sistema, a função de transferência que os descreve é frequentemente uma expressão complexa, contendo os componentes inercial e transcendental.

Problema: A natureza elástica e distributiva dos parâmetros do sistema torna o controle preciso de tensão e velocidade síncrona muito mais complicado do que o sistema de parâmetros centralizado.

Metodologia: Propomos uma solução numérica para sintetizar os reguladores com base no método de interpolação real para reduzir a capacidade computacional e o erro de síntese, preservando as propriedades características de objetos com parâmetros distribuídos.

Conclusão: A eficácia do algoritmo proposto é verificada por um modelo experimental do sistema de acionamento elétrico de dois motores contendo um transportador elástico. A simulação e os resultados experimentais indicam que o sistema de controle com os reguladores recebidos opera de forma estável e atende aos critérios de qualidade exigidos.

Originalidade: Os resultados da pesquisa podem ser aplicados no desenvolvimento de sistemas centrais de controle e monitoramento para linhas de produção automáticas com sistemas de acionamento multimotor que incluem transportadores.

Palavras-chave: Sistema de controle automático, Síntese de reguladores, Objetos com parâmetros distribuídos, Transportador elástico, Função de transferência.

1. INTRODUCTION

Multi-motor electric drive systems (Figure 1) are essential components in many production lines, including industrial robots, mechanical processing equipment (such as continuous rolling mills), paper production lines, optical thin-film handling equipment, cable sheathing lines, spinning machines, canning lines, and automatic robots in automobile assembly lines, as well as in the production of electronic components. The control quality of these systems is critical to the accuracy and overall work quality of the entire production line. The system parameters, such as mass and elasticity of the conveyor, depend on its spatial dimensions and are described by complex integro-differential equations and other mathematical formulations. In general, the transfer function describing this element will have the form:

$$W_{dt}(s) = f(s, e^{\frac{A(s)}{B(s)}}, \sqrt{s}, cos\ cos\ (s)\ , sin\ sin\ (s)\ , sh(s), ch(s), \ldots) \qquad (1)$$

containing not only the argument as s but also the functions of s ($\sqrt{s}, cos\ cos\ (s)\ , sin\ sin\ (s)\ , sh(s), ch(s), \ldots$) [1-4]. This complicates the synthesis of control systems.

The design of a controller for a multi-motor drive system connected together by an elastic conveyor must meet the following requirements: synchronize the motor speeds with high accuracy, and concurrently adjust both speed and torque to stabilize the tension of the conveyor belt.

Some studies have carried out modeling and construction of controllers for multi-motor drive systems containing elastic conveyors [5-11]. There are various control methods developed utilizing classical control theory, fuzzy logic, neural networks, and sustainable optimal control, etc [12-26].

The classical methods synthesize the regulator based on the analysis of characteristics in the time or frequency domains. However, this approach involves complex models and algorithms, a high computational volume and large errors. For instance, when using frequency models to manipulate functions with imaginary arguments, jω, the method is only suitable for linear systems. The synthesis of regulators based on example standard H^{∞} can lead to unsustainable solutions. The limitation of using the integral asymptote criterion between the desired and synthesized system also exists in some methods. Methods based on fuzzy theory and neural networks often require heavy dependence on the designer's experience and involve high computational costs due to the mathematical tools used. The Linear Quadratic Gaussian (LQG) optimal control method requires complete information about the system parameters, which can be difficult to obtain in complex multi-motor systems. However, there are other control methods for multi-motor drive systems, such as adaptive control, sliding control, decentralized control, feedback control based on observers, and other approaches and studies.

Based on the above analysis, this study proposes a solution to synthesize and calibrate the regulator using the real interpolation method. This method has a simple procedure and allows for a reduction in computational volume, while preserving the specific features and effects of conveyors on the system [33-35].

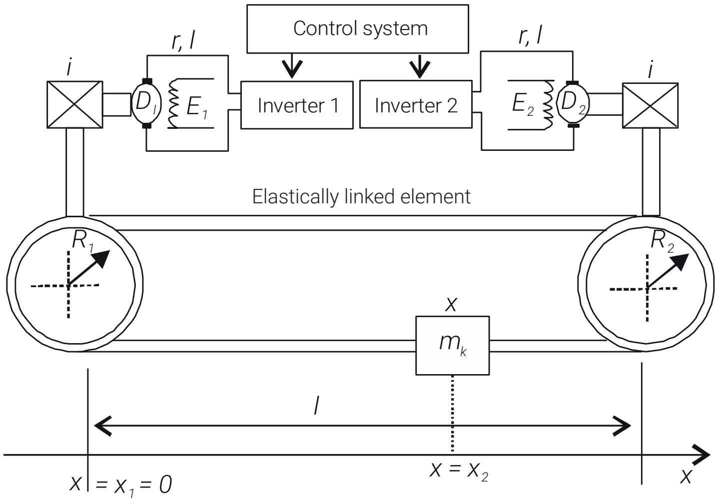


Figure 1. The structure of an active two-motor drive system linked by elastic conveyors.

Source: own work

2. MATERIALS AND METHODS

Section 2.1 presents the working principle of an electric drive system with an elastic conveyor. Our proposed approach to synthesizing the controllers is outlined in Section 2.2, followed by the implementation details in Section 2.3.

2.1. Modeling an electric drive system with elastic conveyor

The elastically linked conveyor is surveyed as a closed-loop form associated with the motor and has the calculation diagram shown in Figure 1. This element has a uniformly distributed mass and stiffness, with m_1 - the mass of the drive part associated with the motor, concentrated at the point $x = x_1 = 0$, also m_2 - the mass of the conveyor is concentrated at the point $x = x_2$. Its mathematical description has the form [2,3]:

$$L_t[u(x,t) = \rho(x)\frac{\partial^2 u(x,t)}{\partial t^2} - E\frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t); 0 \le x \le l, t \ge 0, \rho(x) > 0, E > 0 \quad (2)$$

$$\{u(x,t)|_{t=0} = u_0(x); \frac{\partial u(x,t)}{\partial t}]_{t=0} = u_1(x)\, u(x,t)|_{x=0} = u(x,t)|_{x=l}; \frac{\partial u(x,t)}{\partial t}]_{x=0} = \frac{\partial u(x,t)}{\partial t}]_{x=l}, \quad (3)$$

in which: L_t – the differential operator; $u(x,t)$ - displacement of the point on the conveyor with coordinates x at the t some time; $E = const$ - the elastic modulus of the element under investigation; $u_0\ (x)\ \ u_1\ (x)$- displacement and displacement rate of the conveyor section at coordinates x and time $t = 0$; $f\ (x,t)$- input impacts in space and time; $\rho(x)$ - the material density of the conveyor in coordinates x, which can be calculated through the mass components m_i corresponding to the coordinates x_i according to the expression:

$$\rho(x) = \rho_l + \sum_{i=1}^{n} m_i\delta(x - x_i), \quad (4)$$

with: $\rho_l = const$ is the density of the conveyor when unloaded. Research [3] shows that the normalized transfer function corresponding to (2) is the solution of the equation:

$$\{L[\widetilde{W}] = [1 + \sum_{i=1}^{n} \eta_i\delta(x - x_i)]\tilde{s}^2\widetilde{W}(x,\xi,\tilde{s}) - \frac{d^2\widetilde{W}(x,\xi,\tilde{s})}{dx^2}\delta(x - \xi), \tilde{s} = s/a\, \widetilde{W}(0,\xi,\tilde{s}) = \widetilde{W}(l,\xi,\tilde{s}); \widetilde{W} = aW, a^2 = E/\rho_l\ \frac{d\widetilde{W}(0,\xi,\tilde{s})}{dx} = \frac{d\widetilde{W}(l,\xi,\tilde{s})}{dx}; 0 \le \xi \le l; \eta_i = m_i/\rho_l \quad (5)$$

The survey system (Figure 1) is converted to a two-mass system associated with the motor and conveyor: $n = 2$ [4]. The output point has coordinates x_2 and mass m_2, and the motor is located at the point with coordinates $x_1 = \xi = 0$. The research [2,3] shows that the transfer function representing the relationship between the force at a point of coordinate $x_1 = 0$ and mass m_1 $(F_d(0,s))$, with the velocity at the point of coordinate x_2 and mass: m_2 $(V_\lambda(x_2,s))$

$$W_{dt}(s) = W(x_{2,}0,s) = \frac{V_\lambda(x_2,s)}{F_d(0,s)} = \frac{q.sh.sch\lambda.s}{sh^2s+\mu_1\mu_2s^2(ch^2s-ch^2\lambda s)+(\mu_1+\mu_2)s.sh2s'} \qquad (6)$$

where, $\mu_1 = \eta_1/l$=m_1/m_k, $\mu_2 = \eta_2/l$=m_2/m_k,$m_k = \rho_l l$ - the mass of conveyor; $q = 1/2a$ - the transmission coefficient of the conveyor, $\lambda = 1 - 2x_2/l$ - the output space coordinates of the system. The calculation is done in a similar way for input-output coordinates: $x_1 = \xi = 0, x_2 = 0$, and we will get the transfer function of the relationship between the force on the active rewinder $(F_d(0,s)$and its velocity $(V_d(0,s)$:

$$W^*{}_{dt}(s) = W(0_,0,s) = \frac{V_d(0,s)}{F_d(0,s)} = \frac{q[sh2s+s\mu_2(ch^2-ch^2\lambda s)}{sh^2s+\mu_1\mu_2s^2(ch^2s-ch^2\lambda s)+(\mu_1+\mu_2)s.sh2s} \qquad (7)$$

Based on the description of the force channel of the inverter-asynchronous electric motor in the rotation coordinate system $(d,\ q)$ oriented to the total flux vector of the rotor, the general structure diagram of the electric drive system, taking into account the influence of elastic conveyor (7) fed into the feedback circuit of the speed control loop, will have the form shown in Figure 2.

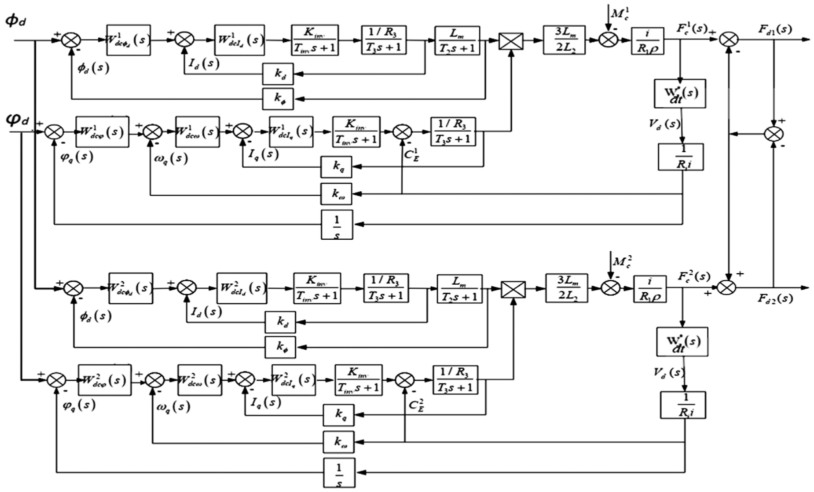


Figure 2. Structure diagram of two asynchronous motor electric drive systems taking into account the influence of elastic conveyor
Source: own work

(K_{inv}, T_{inv} - the parameters of the inverter; k_d - Coefficient of current-loop feedback; , $R_3 = R_1 + R_2 \frac{L_m^2}{L_2^2}$ in which: R_1 - Resistance of the stator circuit; L_m - The total inductance is created by the magnetic flux in the motor gap; L_2 - The equivalent inductance of the rotor coil; R_2 - The stator conversion resistance of the Rotor circuit; $T_3 = \frac{\sigma L_1}{R_3}$ in that: σ - Total dissipation factor; L_1 - Equivalent inductance of the stator coil).

2.2. Synthesis of controllers

In practice, the synthesis of control systems is usually done in two approaches:

- The first approach: we consider the system as a collection of separate control loops that do not affect each other. In this case, each loop is synthesized separately from the inner loop to the outer loop [2]. The limitation of this approach is that the accumulated error increases with each iteration because we only know in advance the desired properties of the whole system, while information about the desired properties of each inner loop is needed for designing its controller. Therefore, it is necessary to determine the desired properties for each loop. However, it is impossible to have an exact solution, leading to increased errors throughout the synthesis.
- The second approach: we set up a general synthesis equation, containing the desired coefficients of all the regulators. It leads to nonlinear equations for the parameters of the regulator. Such a synthesis method requires the expansion of the original equation into a system of nonlinear equations. Solving these systems will be difficult to implement in the Time, Fourier or Laplace domains [3,4].

From the above analysis, this study will consider the first approach. With the structure diagram in Figure 2, the control system for each motor will include two control loops: flux and speed.

2.2.1. Synthesis of magnetic flux control loop

The flux control loop consists of a current loop and a flux loop. The synthesis equation of the current loop will form:

$$W_{mI_d}^{k}(s) \cong \frac{W_{dcId}^{1}(s).\frac{K_{inv}}{T_{inv}s+1}.\frac{1/R_3}{T_3 s+1}}{1+k_d.W_{dcId}^{1}(s).\frac{K_{inv}}{T_{inv}s+1}.\frac{1/R_3}{T_3 s+1}}. \quad (8)$$

The task is to determine if the regulator $W^{1}_{dcI_d}(s)$ has the general form:

$$W^{1}_{dcI_d}(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_1} (m < n), \tag{9}$$

and the feedback coefficients k_d so that the system satisfy the condition:

$$\{ \sigma_R - \Delta\sigma \leq \sigma_s \leq \sigma_R + \Delta\sigma \; t^{S}_{ST} \leq t^{R}_{ST} \, , \tag{10}$$

or

$$\{ \sigma_R - \Delta\sigma \leq \sigma_s \leq \sigma_R + \Delta\sigma \; t^{S}_{ST} \to t^{min}_{ST} \, , \tag{11}$$

with, $W^{k}_{mI_d}(s)$ - the desired transfer function of the current loop; σ_R - the overshoot of the desired system; σ_s - the overshoot of the synthesized system; $\Delta\sigma$ - the allowable overshoot erroneous ($\Delta\sigma = |\sigma_s \leq \sigma_R|$); t^{R}_{ST} - the settling time of the desired system; t^{S}_{ST}, t^{min}_{ST} - the settling time and the minimum achievable settling time of the synthesized system, respectively. In fact, when there is no regulator (9) for the system to satisfy the condition (10), we can move to condition (11), where the overshoot complies with the tight limit and also the excessive time is minimal (which is still larger than the required value $t^{min}_{ST} > t^{R}_{ST}$). The transition from condition (10) to (11) ensures that the synthesis problem always has a solution without losing its generality.

To solve the equation (8), it is first necessary to determine the function $W^{k}_{mI_d}(s)$ according to the required quality criteria σ_R, t^{R}_{ST}. One of the methods developed by А.С. Коновалов and И.А.Орурк [33], determining the function $W^{k}_{mI_d}(s)$ has the form

$$W^{k}_{mId}(s) = \frac{\frac{\alpha_1}{2} s + 1}{\alpha_0 s^2 + \alpha_1 s + 1} H^{\infty}_{R}; \alpha_0 = \frac{[ln ln \left(\frac{H^{max}}{H^{\infty}_{R}} - 1 \right)]^2}{\frac{9}{(t^{R}_{ST})^2} \{ \left[ln ln \left(\frac{H^{max}}{H^{\infty}_{R}} - 1 \right) \right]^2 + \pi^2 \}}; \alpha_1 = \frac{6\alpha}{t^{R}_{ST}}, \tag{12}$$

with: H^{∞}_{R}, H^{max} - the steady-state value and the corresponding maximum of the step response determined based on the required overshoot σ_R and the desired static mode of the system. When $\sigma_R = 4\%; t^{R}_{ST} = 0.08(s); H^{\infty}_{R} = 1$, the expression (12) takes the form:

$$W^{k}_{mId}(s) = \frac{0.012s + 1}{2.4 * 10^{-4} s^2 + 2.4 * 10^{-2} s + 1}. \tag{13}$$

Based on the structure diagram (Fig. 2) and the specifications of the motor (GA-28), inverter (Siemens G120) and conveyor, we can determine the parameters: Motor: $L_1 = 0.3135(H)$; $L_2 = 0.3178(H)$; $\sigma = 0.0903$; $L_m = 0.301(H)$; $R_3 = 6.602(\Omega)$, $T_3 = 0.00429\ (s)$; $T_2 = 0.09287\ (s)$; $Z_p = 2$; $J_s = 0.033$; Inverter: output voltage $U_{đm} = 230(V)$, control voltage $U_{đk} = 10(V)$, switching frequency $f = 500(Hz)$, $K_{inv} = 31.1$; $T_{inv} = 0.625 * 10^{-4}(s)$; Conveyor: $q = \frac{1}{2a} = 7, \lambda = 1 - \frac{2x_2}{l} = 0.4, \mu_1 = \frac{m_1}{m_k} = 11, \mu_2 = \frac{m_2}{m_k} = 0; \rho = 1; R = 1; i = 4$, and current regulator of the form PI:

$$W^1_{dcI_d}(s) = K_p^d + \frac{K_I^d}{s}, \tag{14}$$

relation (8) has an explicit form:

$$\frac{0.012s+1}{2.4*10^{-4}s^2+2.4*10^{-2}s+1} = \frac{\frac{K_p^d s+K_I^d}{s}\cdot\frac{31.1}{0.625*10^{-4}s+1}\cdot\frac{0.15}{4.29*10^{-3}s+1}}{1+k_d\frac{K_p^d s+K_I^d}{s}\cdot\frac{31.1}{0.625*10^{-4}s+1}\cdot\frac{0.15}{4.29*10^{-3}s+1}} \tag{15}$$

The essence of the synthesis method applying the real interpolation method is to convert the synthetic equation (15) to the form with real arguments [33-35], choose the distribution rule of interpolated nodes δ_i, for example:

$$\delta_i = i\delta_1, i = 1 \div \eta, \tag{16}$$

with η – the number of nodes or establishment the nodes $\{\delta_i\}_n$ to coincide with the zero points of a Chebyshev polynomial of the first order of degree η $(T_n(x))$ to increase the total accuracy [36]:

$$\delta_i = \frac{1+x_i}{1-x_i}a, i = \underline{1,\eta} \tag{17}$$

with: a– The real parameter is used to correct the synthesis error, and $\{x_i\}_n$ – roots of equation: $T_n(x) = 0$, which is determined by the relation [36]:

$$\{T_0(x) = 1\ T_1(x) = x; T_2(x) = x^2 - \frac{1}{2}\ T_{\eta+1}(x) = xT_\eta(x) - \frac{1}{4}T_{\eta-1}(x); x \in [-1,1]\,. \tag{18}$$

Note that the feedback coefficients k_d can be determined from the condition:

$$k_d < \frac{1}{H_d^{\infty} - \Delta H_d} = \frac{1}{H_d^{\infty}}, \tag{19}$$

with, ΔH_d - error in the settling mode.

Solving equation (15) with two interpolated points (δ_1, δ_2) in the interval $\delta \in [0.1,2]$, and the interpolation step: $\Delta\delta = 0.1$, we get the results in Table 1.

Table 1. Synthetic results of the magnetic flux control loop

Loop	**interpolation interval**	**Regulator**	**Quality criteria**					
			Desired criteria			**Synthesis criteria**		
			$\sigma_R(\%)$	$t_{ST}^{R}(s)$	H_R^{∞}	$\sigma_S(\%)$	$t_{ST}^{S}(s)$	H_S^{∞}
Current	$[0.1,2]$	$\frac{-0.044s + 11.87}{s}$	4	0.08	1	1.48	0.036	1
Magnetic flux	$[0.1,2]$	$\frac{7.29s + 85.73}{s}$	4	0.15	1	3.4	0.15	1

Source: own work

The synthetic equation of the flux loop containing the current loop is expressed by the relation:

$$W_{m\emptyset_d}^{k}(s) \cong \frac{W_{dc\emptyset_d}^{1}(s) W_{vd}(s) \frac{Lm}{T_2 s+1}}{1 + k_{\emptyset} W_{dc\emptyset_d}^{1}(s) W_{vd}(s) \frac{Lm}{T_2 s+1}} \tag{20}$$

with: $T_2 = \frac{L_2}{R_2}$; $W_{dc\emptyset_d}^{1}(s)$; - flux regulator, then the transfer function of the current loop $W_{vd}(s)$ with regulator (14) is determined by the expression:

$$W_{vd}(s) = \frac{\frac{-0.044s+11.87}{s} \cdot \frac{31.1}{0.625*10^{-4}s+1} \cdot \frac{0.15}{4.29*10^{-3}s+1}}{1 + \frac{-0.044s+11.87}{s} \cdot \frac{31.1}{0.625*10^{-4}s+1} \cdot \frac{0.15}{4.29*10^{-3}s+1}} \tag{21}$$

The desired transfer function $W_{m\emptyset_d}^{k}$ with the given request parameters: $\sigma_R = 4\%; t_{ST}^{R} = 0.15(s); H_R^{\infty} = 1$ will form:

$$W_{m\emptyset_d}^{k}(s) = \frac{0.022s+1}{8*10^{-4}s^2 + 0.044s + 1}. \tag{22}$$

Then execute solving synthesis equation (20) in the same way as the current loop synthesis with interpolation interval: $\delta \in [0.1,2]$; interpolation step: $\Delta\delta = 0.1$; feedback factor: $k_{\emptyset} = 1$, we receive flux regulators form PI:

$$W^{1}_{dc\phi_d}(s) = \frac{7.29s+85.73}{s}, \tag{23}$$

and the quality criteria of the post-synthesized magnetic flux ring are listed in Table 1.

2.2.2. Synthesis of speed control loop

Synthesis equation for speed control loop:

$$W^{k}_{m\omega}(s) \cong \frac{W^{1}_{dc\omega}(s)W_{vd}(s)\frac{3L_m\ \ i}{2L_2\ R_1\rho}W^{*}_{dt}(s)\frac{1}{iR_1}}{1+k_\omega W^{1}_{dc\omega}(s)W_{vd}(s)\frac{3L_m\ \ i}{2L_2\ R_1\rho}W^{*}_{dt}(s)\frac{1}{iR_1}}, \tag{24}$$

with: $W^{k}_{m\omega}(s)$ - the desired transfer function of speed loop; $W^{1}_{dc\omega}(s)$ - the speed regulator; $W_{vd}(s)$ - the transfer function of the current loop has the form (45); $W^{k}_{m\omega}(s)$ - the desired transfer function of speed loop, with given request parameters: $\sigma_R = 4\%$; $t^{R}_{ST} = 0.5(s)$; $H^{\infty}_{R} = 3$, will form:

$$W^{k}_{m\omega}(s) = \frac{0.2265s+3}{9.56*10^{-3}s^2+0.15s+1}. \tag{25}$$

The solution of equation (24) is done in the same way as the current loop synthesis with interpolation interval: $\delta \in [0.1,2]$; interpolation step: $\Delta\delta = 0.01$; feedback factor: $k_\omega = 1$, we receive the speed regulator form PI:

$$W^{1}_{dc\omega}(s) = \frac{55.79s+8.68}{s}, \tag{26}$$

and the quality criteria of the post-synthesized system are displayed in Table 2.

Table 2. Synthetic results of speed control loop

Loop	**interpolation interval**	**Regulator**	**Quality criteria**					
			Desired criteria			**Synthesis criteria**		
			$\sigma_R(\%)$	$t^{R}_{ST}(s)$	H^{∞}_{R}	$\sigma_S(\%)$	$t^{S}_{ST}(s)$	H^{∞}_{S}
Speed	$[0.1,2]$	$\frac{55.79s+8.68}{s}$	4	0.5	3	1	0.36	3
Rotation angle	$[0.1,2]$	6.73	12	1	6	1.6	0.66	6

Source: own work

The synthesis equation for the rotation angle control loop is represented as:

$$W_{m\varphi}^{k}(s) \cong \frac{W_{dc\varphi}^{1}(s)W_{vtd}(s)}{1+k_{\varphi}W_{dc\varphi}^{1}(s)W_{vtd}(s)}, \tag{27}$$

in which, $W_{dc\varphi}^{1}(s)$ – is the angle of rotation, then transfer function of the speed loop $W_{vtd}(s)$ is determined by the expression:

$$W_{vtd}(s) \cong \frac{W_{dc\omega}^{1}(s)W_{vd}(s)\frac{3Lm\quad i}{2L_2\, R_1\rho}W_{dt}^{*}(s)\frac{1}{iR_1}}{1+k_{\omega}W_{dc\omega}^{1}(s)W_{vd}(s)\frac{3Lm\quad i}{2L_2\, R_1\rho}W_{dt}^{*}(s)\frac{1}{iR_1}}. \tag{28}$$

Desired transfer function of position loop $W_{m\varphi}^{k}(s)$, with given request parameters: $\sigma_R = 12\%; t_{ST}^{R} = 1(s); H_R^{\infty} = 6$, sẽ có dạng:

$$W_{m\varphi}^{k}(s) = \frac{0.68s+6}{0.035s^2+0.2268s+1}. \tag{29}$$

Then execute solving synthesis equation (28) in the same way as the current loop synthesis with interpolation interval: $\delta \in [0.1,\, 2]$; interpolation step: $\Delta\delta = 0.01$; feedback factor: $k_{\varphi} = 1/s$, we get an angle regulator (P):

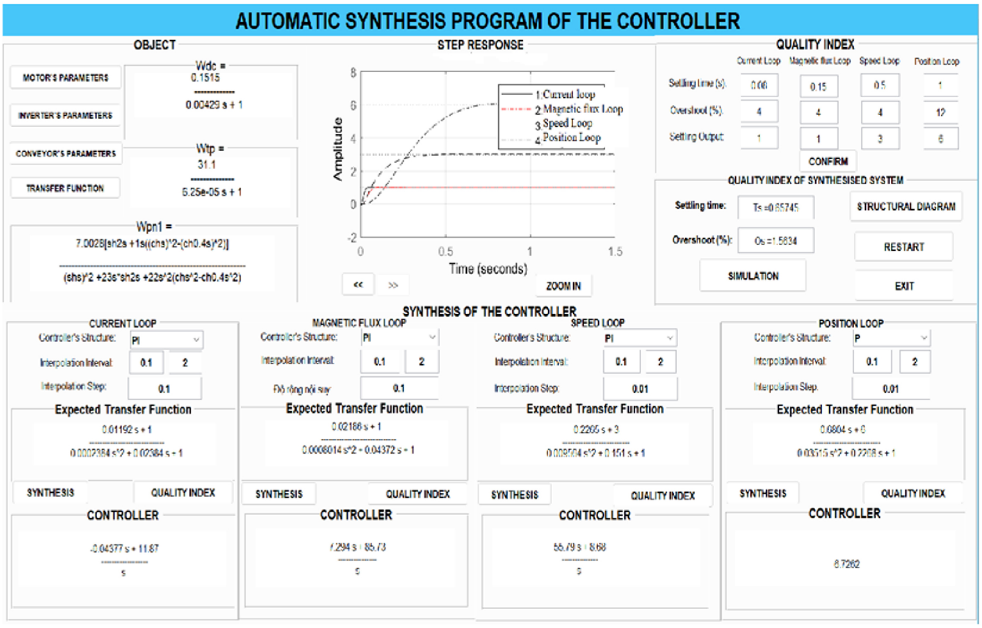
$$W_{dc\varphi}^{1}(s) = 6.73, \tag{30}$$

and the quality criteria of the post-synthesized system are listed in Table 2.

2.3. Synthesis program

From the analysis in Section 2.2, synthesis procedure is presented in algorithm:

1. Enter the input data: motor, inverter, conveyor and requirements of the system and each loop: $\sigma_R, t_{ST}^{R}, \sigma_R^{i}, t_R^{i}$.
2. Identify the desired function by (12) and determine the feedback coefficients of each loop by (19).



Wdc =
0.1515

0.00429 s + 1

Wtp =
31.1

5.25e-05 s + 1

Wpn1 =
7.0028[sh2s +1s((chs)^2-(ch0.4s)^2)]

(shs)^2 +23s*sh2s +22s^2(chs^2-ch0.4s^2)

QUALITY INDEX

	Current Loop	Magnetic flux Loop	Speed Loop	Position Loop
Settling time (s):	0.06	0.15	0.5	1
Overshoot (%):	4	4	4	12
Setting Output:	1	1	3	6

QUALITY INDEX OF SYNTHESISED SYSTEM

Settling time: Ts =0.85745

Overshoot (%): Os =1.5834

CURRENT LOOP

Controller's Structure: PI

Interpolation Interval: 0.1 2

Interpolation Step: 0.1

Expected Transfer Function

0.01192 s + 1

0.0002384 s^2 + 0.02384 s + 1

CONTROLLER

-0.04377 s + 11.87
----
s

MAGNETIC FLUX LOOP

Controller's Structure: PI

Interpolation Interval: 0.1 2

Độ rộng nội suy 0.1

Expected Transfer Function

0.02186 s + 1

0.0006014 s^2 + 0.04372 s + 1

CONTROLLER

7.294 s + 85.73
----
s

SPEED LOOP

Controller's Structure: PI

Interpolation Interval: 0.1 2

Interpolation Step: 0.01

Expected Transfer Function

0.2265 s + 3

0.008504 s^2 + 0.151 s + 1

CONTROLLER

55.79 s + 8.68
----
s

POSITION LOOP

Controller's Structure: P

Interpolation Interval: 0.1 2

Interpolation Step: 0.01

Expected Transfer Function

0.6804 s + 6

0.03515 s^2 + 0.2268 s + 1

CONTROLLER

8.7262

Figure 3. Program interface for the automatic synthesis of a two-motor electric drive system containing an elastic conveyor

Source: own work

3. Select the regulator structure of each loop (m,n) by (9) and identify interpolation node points δ_i by (16) or (17)
4. Set up and solve equation (8), (20), (24), (27) for each loop. Determining the quantities (σ_S, t_{ST}^S) of the synthesized system.
5. If the system does not satisfy conditions (10) or (11), then return to step 3.

The main interface of the synthesis program written on Matlab 2017b is shown in Figure 3. The system simulation diagram on Simulink is shown in Figure 4. Step responses of the synthesized system in each loop are shown in Figure 3, while the tension variation at the motor shafts $(x = 0)$ is shown in Figure 5.

The calculation results in Tables 1,2 show that the received regulators meet the required quality criteria of the surveyed system, ensuring stability and synchronous speed tracking of the motors in both transient mode and steady mode. The graphs in Figure 5 show that the tension of the conveyor belt generated by the speed difference between the motors is negligible.

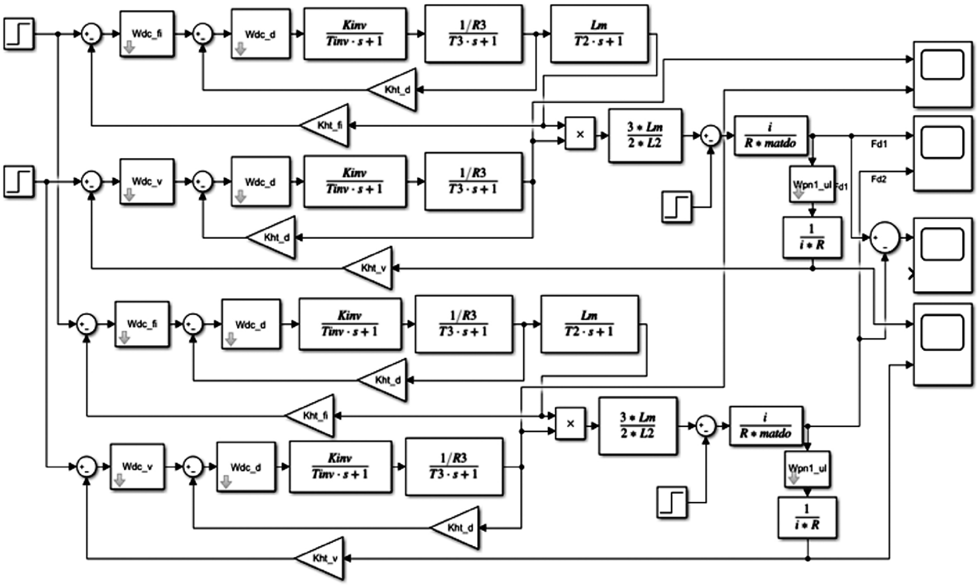


Figure 4. Structural diagram of the system on Simulink
Source: own work

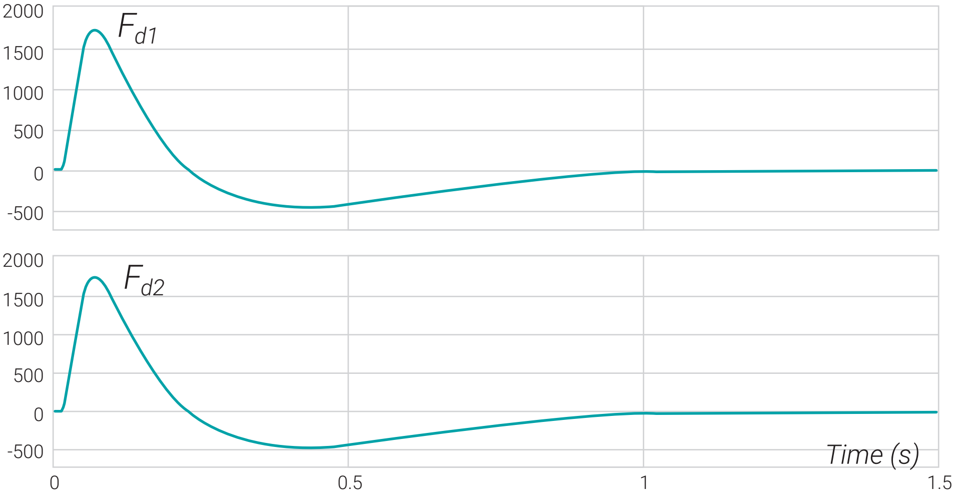


Figure 5. Tension variation of conveyor at two motor shafts (F_{d1}, F_{d2})
Source: own work

3. Experimental results

To verify the proposed method, an experimental model of the drive control system of two asynchronous motors linked together by an elastic conveyor is built. These motors have a three-phase squirrel cage rotor (GA-28) and Siemens G120 inverter.

The parameters of the current regulator (14) and the magnetic flux (23) are set while setting the inverter by the program STARTER. The parameters of the speed controller (26) are set by the PIDs of the PLC stations. The experimental model is shown in Figure 6.

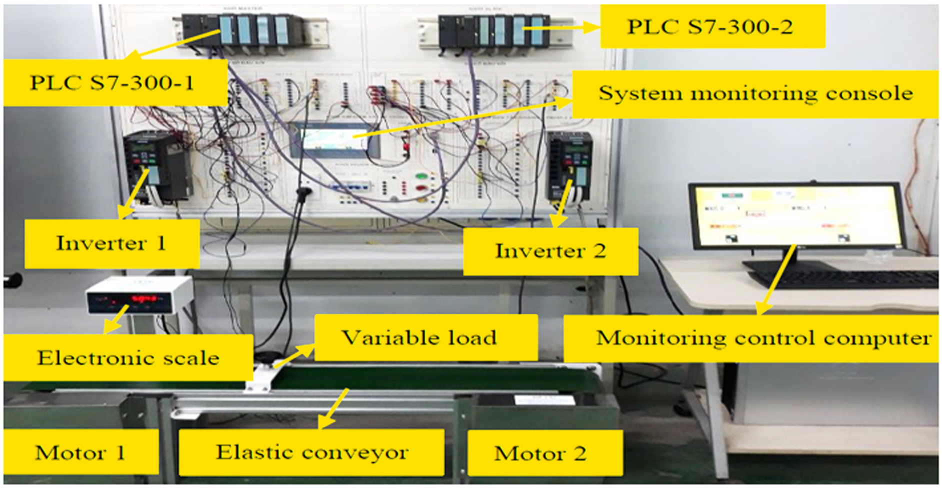
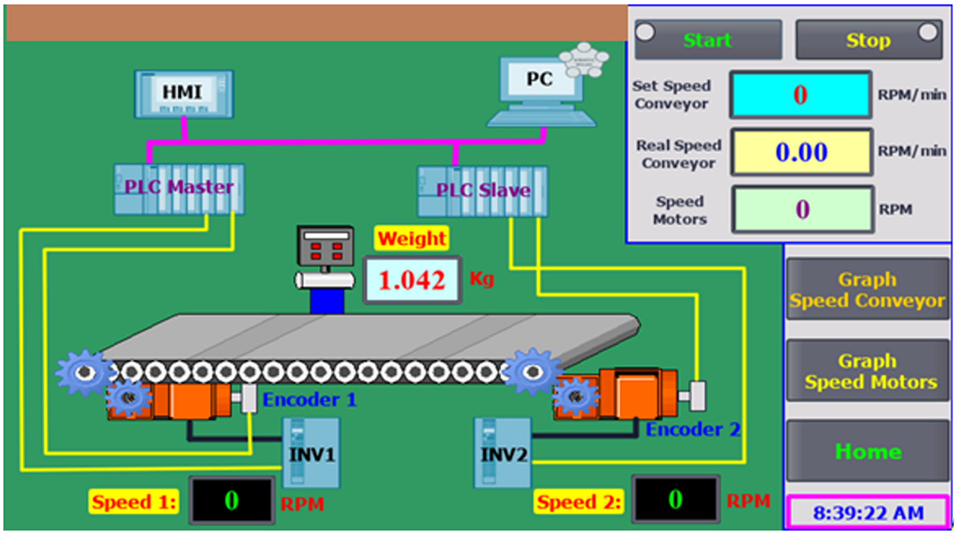


Figure 6. Experimental model of transmission control of two asynchronous motors linked by elastic conveyor

Source: own work

The control program is written in “Step7,” while the system monitoring program is written in “WinCC.” The interface is displayed in Figure 7.



Set Speed Conveyor 0 RPM/min
Real Speed Conveyor 0.00 RPM/min
Speed Motors 0 RPM
Weight 1.042 Kg
Speed 1: 0 RPM
Speed 2: 0 RPM
8:39:22 AM

Figure 7. Interface of the control and monitoring system on the computer

Source: own work

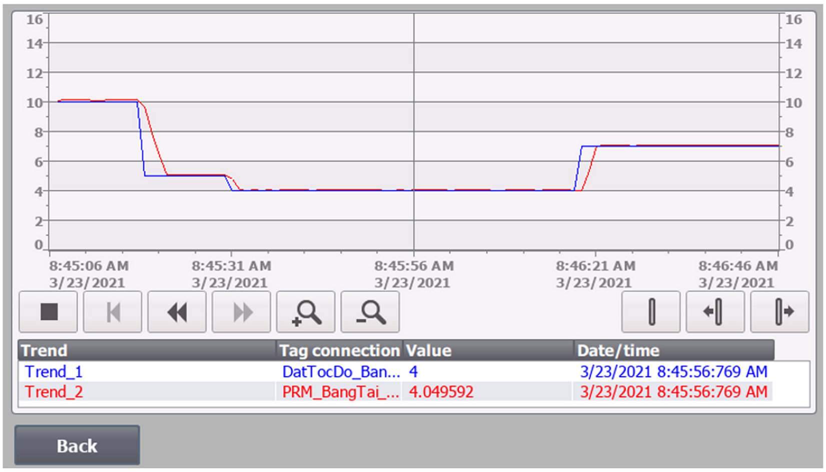
Table 3 presents the experimental results obtained when the load was varied from 0 to 10 kg, and the conveyor deflection was varied from 0 to 1 cm. Some conclusions can be drawn from testing the device in different modes:

- The speed of the two motors are stable according to the set value and trace to each other. When the load changes, the motor speed also changes but quickly stabilizes to the set value.
- When the load changes, the speed of each motor decreases compared to the set speed but not significantly. Conveyor tension deviations at the two motor axes can be neglected.
- The characteristic of speed tracking in Figure 8 shows that the system responds quickly to control signals and load changes, and the delay in motor speeds is negligible.

Table 3. The experimental results were obtained by varying the load from 0-10 kg and the conveyor deflection from 0-1 cm

Numbers	Working Range (rpm)	Static error (%)	Settling time (s)	Overshoot (%)
Preset speed	362			
Motor speed 1	365	0.8	2	0.1
Motor speed 1	364.5	0.7	2	0.1

Source: own work



Trend	Tag connection	Value	Date/time
Trend_1	DatTocDo_Ban...	4	3/23/2021 8:45:56:769 AM
Trend_2	PRM_BangTai_...	4.049592	3/23/2021 8:45:56:769 AM

Figure 8. Conveyor speed variation chart from 10m/min -> 5m/min -> 4m/min -> 7m/min when the load changes from 0-10 kg

Source: own work

4. DISUSSIONS AND CONCLUSIONS

The analysis of the experimental results indicates that the control system effectively maintains the stability and speed synchronization requirements of the motors in both transient and settling modes. The tension deviation of the conveyor at the motor shafts, resulting from the speed difference between the motors, is negligible. Moreover, the experimental system exhibits overshoot and settling times that are close to the simulation results. The above experimental model has the advantage of not needing tension sensors (load cells) because the influence of the conveyor has been introduced into the speed loop through the transfer function $W_{dt}^{*}(s)$ (Figure 2).

The method considered in the paper provides a feasible way to solve the problem of synthesizing regulators of a multi-motor electric drive system containing an elastic conveyor. The calculation and simulation results show the accuracy of the proposed method, which allows direct manipulation of the original model describing the conveyor (10) without any significant difficulties.

However, the actual test shows that the system will be unstable when the load changes significantly (the parameters of the conveyor will change accordingly). Therefore, future work includes:

Improving the working quality of the multi-motor drive control system linked by the elastic conveyor in the case of large load changes by updating the parameters of regulator for each range of load variation.

Also, the method described above can be applied to synthesize other specific systems with distributed parameters, such as heating processes and towed underwater vehicles. This will enable a more comprehensive evaluation of the effectiveness of the proposed method.

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