A Differential Cooperative Positioning Approach for Multi-Device Positioning Improvement

Una metodología de posicionamiento cooperativo diferencial para el posicionamiento de dispositivos múltiples

Uma metodologia de posicionamento cooperativo diferencial para o posicionamento de dispositivos múltiplos

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Abstract

Introduction: This publication is the product of research developed within the research lines of the Advanced and Large-scale Computing (CAGE) research group throughout 2018, which supports the work of a master's degree in Systems Engineering at the Industrial University of Santander.

Objetive: An approach to a cooperative positioning algorithm is described in this paper, where a set of devices exchange GPS satellite observables and distance estimations with nearby devices in order to increase their positioning accuracy.

Methodology: Different scenarios are established where GPS receivers exchange satellite information, using different ionospheric correction models, with the purpose of evaluating which conditions potentially improve the position accuracy.

Conclusions: The results show our approach yields increased accuracy when all receivers use the same ionospheric correction model. Moreover, it was observed that the noise levels and uncertainty usually due to factors related to distance from remote devices to the main receiver did not influence positioning improvement when the separation between receiver pairs was large.

Originality: The proposed algorithm allows for exploitation of the nature of the problem without increasing complexity at the hardware and software level, and to offer a low-cost cooperative positioning solution alternative.

Restrictions: The results presented in the document are based on the execution of the cooperative algorithm using RINEX files of GNSS reference stations. So, for scenarios in which the separation distances between reference stations are very high, the error levels in cooperative positioning can be very large.

Keywords: Cooperative Positioning, Differential Positioning, Weighted Least Squares, Single Frequency GPS Receivers, Matrix Of Direction Cosines.

Resumen

Introducción: esta publicación es el producto de una investigación del grupo de investigación de computación avanzada y en gran escala (CAGE) de la Universidad Industrial de Santander, a lo largo de 2018.

Objetivo: Se propone un algoritmo de posicionamiento cooperativo en el que un conjunto de dispositivos intercambia observables satelitales, y estimaciones de distancia entre dispositivos GPS cercanos, con el objetivo de aumentar su precisión de posicionamiento.

Metodología: se establecen escenarios donde los receptores de GPS intercambian información satelital, y utilizan diferentes modelos de corrección ionosférica con el fin de evaluar las condiciones en que es posible mejorar la precisión en posicionamiento.

Conclusiones: El algoritmo propuesto produce una mayor precisión cuando todos los receptores emplean el mismo modelo de corrección ionosférica. Además, el nivel de incertidumbre en la medida de distancia entre dispositivos no presenta mayor influencia sobre la mejora de la precisión, cuando la separación entre receptores es muy grande.

Originalidad: el algoritmo propuesto permite explotar la naturaleza del problema sin aumentar la complejidad a nivel de hardware y software, y se ofrece como una alternativa de solución de posicionamiento cooperativo de bajo costo.

Limitación: Los resultados exponen la ejecución del algoritmo cooperativo utilizando archivos RINEX de estaciones de referencia GNSS. Por lo tanto, para los escenarios en que la distancia de separación entre estaciones es muy alta, los niveles de error en posicionamiento pueden ser elevados.

Palabras clave: posicionamiento cooperativo, posicionamiento diferencial, mínimos cuadrados ponderados, receptores GPS de una sola frecuencia, matriz de cosenos de dirección.

Resumo

Introdução: esta publicação é o produto de uma pesquisa do grupo de pesquisa de computação avançada e em grande escala (Cage) da Universidad Industrial de Santander (Colômbia), durante 2018.

Objetivo: propõe-se um algoritmo de posicionamento cooperativo no qual um conjunto de dispositivos intercambia observáveis de satélites e estimativas de distância entre dispositivos gps próximos, com o objetivo de aumentar sua precisão de posicionamento.

Metodologia: são estabelecidos cenários onde os receptores de gps trocam informação de satélites e utilizam diferentes modelos de correção ionosférica a fim de avaliar as condições em que é possível melhorar a precisão em posicionamento.

Conclusões: o algoritmo proposto produz uma maior precisão quando todos os receptores empregam o mesmo modelo de correção ionosférica. Além disso, o nível de incerteza na medida de distância entre dispositivos não apresenta maior influência sobre a melhora da precisão, quando a separação entre receptores é muito grande.

Originalidade: o algoritmo proposto permite explorar a natureza do problema sem aumentar a complexidade no nível de hardware e software, e se oferece como uma alternativa de solução de posicionamento cooperativo de baixo custo.

Limitação: os resultados expõem a execução do algoritmo cooperativo utilizando arquivos Rinex de estações de referência gnss. Portanto, para os cenários em que a distância de separação entre estações é muito alta, os níveis de erro em posicionamento podem ser elevados.

Palavras-chave: posicionamento cooperativo, posicionamento diferencial, mínimos quadrados ponderados, receptores gps de uma frequência só, matriz de cossenos de direção.

1. Introduction

A great deal of mobile technology applications today requires clear and accurate information about the location of people or objects. Although low-cost receivers integrated into mobile phones somehow supply the location needs, they exhibit high error levels for certain types of applications and/or environments.

On the other hand, satellite positioning systems are characterized by having a unidirectional flow of information. Namely, when the signals travel from the satellite to the receiver, the latter is responsible for collecting all the information necessary to determine its position accurately. Considering that the capacity of satellite receivers for reaching high levels of accuracy is proportional to their cost, the search for new alternatives that increase precision levels without raising per-device cost is of great interest for applications that use global satellite navigation systems (GNSS).

It is known that the precision level for positioning tasks is acceptable for ideal open sky conditions. However, not all environments offer this possibility to receivers; urban environments or areas with high electromagnetic interference are conditions in which the positioning accuracy of GNSS devices is strongly affected.

In this scenario, cooperative positioning emerges as an alternative for GNSS positioning tasks. This paradigm allows GNSS receivers to exchange information, facilitating the improvement of positioning accuracy levels.

2. Literature review

2.1 Satellite Positioning

This section presents concepts related to satellite positioning systems, with the purpose of defining a common framework of reference for the reader. This framework lays down the foundation for our approach to a Differential Cooperative Positioning algorithm supported by Weighted Least Squares method.

2.2 Global Satellite Positioning Systems

The principles behind positioning systems —such as GPS— can be summarized as follows: "If the distance from three satellites in space to a fixed point on the surface of the Earth (a GPS receiver) is known, along with the position of the satellites at the time of the transmission of a satellite to the receiver, the position of the receiver can be determined through the application of trigonometric concepts, algebraics, about a specific coordinate system" [1].

The key issue behind positioning systems is to accurately establish the distance for each satellite. The final objective is to determine the receiver position as precisely as possible. For this purpose, the observational models work as tools to describe the phenomena involved in the signal voyage.

2.3 The Pseudorange

The pseudorange observable is a basic and fundamental concept in GPS. In essence, it is the measured physical range between a satellite and the receiver, including errors attributable to receiver clock bias and error associated with phenomena that cause dispersion and signal delay.

Diversely, when determining the position of a receiver, it is necessary to represent the locus of the satellites taking as reference both the coordinates of satellites and the distance to the position of the receiver. This allows us to find the intersection of the geometric planes of the satellites; as it can be seen in Figure 1.

Besides, the intersection of the geometrical locations of all the visible satellites hardly coincides in a single point, owing to the fact that the signals that travel from the satellite are affected by the dispersion of the ionosphere and other error sources. This generates signal delays and, consequently, errors in the determination of the distance between each satellite and the receiver.

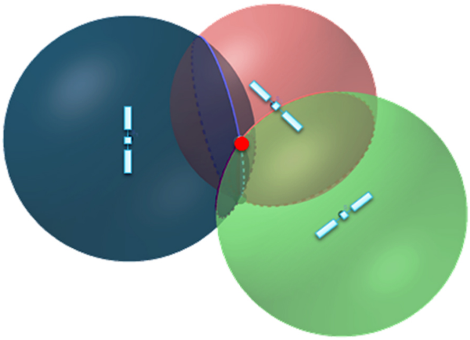


Figure 1. Geometric representation of the concept behind of trilateration.
Source: own work

However, the main source of error in the observable model is the effect caused by an innacurate local oscillator, which causes a synchronization error with respect to satellite system time reference —GPS time—. In order to minimize this error, it is necessary that at least four satellites are visible to the receiver.

The observable value measured by the receiver, can be algebraically described through a physical model such as the time difference between the signal emission and its reception, multiplied by the speed of light.

Some pseudorange models proposed in the literature usually represent the errors associated with orbital satellites as a constant, after considering that the instantaneous change in the position of the satellites is slower with respect to the sampling time of satellite signals at the receiver. In the same way, the ionospheric delay can be modeled using constants or functions with slow exchange rates to represent the variation of the content of electrons from one measurement to another.

These models seek to represent some observables in a better way, looking to express all phenomena in the best possible way, which significantly increases their complexity. This is the case of the generalized equation for the observable pseudo-range, in which most sources of error affecting the satellite signal are taken into consideration in equation (1)

Equation 1 Generalize pseudorange model

$$P_r^s(t) = \rho_r^s(t) + c * \tau_r(t) - c * \tau^s(t) - d_{iono}^s + d_{trop}^s + M_{r,P}^s(t) + \eta \qquad (1)$$

Where:

- $P_r^s(t)$ Is the observable pseudo-range, with all mistakes related to signal adquiring process.
- $\rho_r^s(t)$ Represents the geometrical range between receiver and the satellite in time t.
- c Is the speed of light.
- $\tau_r(t)$ Is the error related to bad time synchronizing in receiver respecting to GPS Time, in time t.
- $\tau^s(t)$ Is the error related to bad time synchronizing in the satellite respecting to GPS Time, in time t.
- d_{iono}^s Is the error related to ionospheric delay.
- d_{trop}^s Is the error related to tropospheric delay.
- $M_{r,P}^s(t)$ Is the error related to the multipath phenomen in time t.
- η Is the error during measurement of the pseudorange $P_r^s(t)$ due to the noise at the receiver, in time t.

Approaches and developments in the literature show that it is possible to support the empirical behavior and characteristics of the phenomena to mitigate the complexity of the observation models. Thus, in some cases, it is possible to represent errors associated with satellite orbits as a constant, when it is considered that the instantaneous change in the position of the satellite is slower compared to the sampling time of signals in the receiver. Similarly, the ionospheric delay can be modeled by constants or functions with slow change rates, representing the variation of the electron content between one measurement or another [2].

A further consideration has to do with the basis for the differential positioning approach, where the ionospheric delay is considered similar between nearby receptors and consecutive measurements (time to period). This is evidenced in the case studies of [3], [4], which show the assumotion that the ionospheric delay for multiple receivers located relatively close to each ($d < 200[km]$), may be applied to all of them.

2.4 Positioning Techniques

Although many of the techniques of positioning make use of the pseudo-range model, not all of them are able to compensate for the effects associated with the error sources.

2.5 Differential Models

Also known as models of baseline positioning. It is a technique that consists of referencing an object with respect to nodes whose location is known. The differential techniques are very common between pairs of receivers when distances between each other do not exceed 200km.

The geometrical representation of the single-difference mathematical model shown in Figure 2., uses the difference between the observable range of two receivers R_{x1}, R_{x2} with respect to a common satellite at the same time —simultaneously—, as shown in the equation (2).

Equation 2 Single Differences between receivers A and B

$$\begin{gathered}\Delta P_{AB}^{s} = P_B^s - P_A^s = (\rho_B^s - \rho_A^s) + c * (\tau_B - \tau_A) + (\epsilon_B^s - \epsilon_A^s) \\ \Delta P_{AB}^{s} = \Delta\rho_{AB}^{s} + c * \Delta(\tau_{AB}) + \Delta(\epsilon_{AB}^{s})\end{gathered} \tag{2}$$

The pseudorange models for receivers A, and B are presented in equation (3) where the terms d_{iono}^{s} and d_{trop}^{s} are assumed to be similar for both receivers, according to what is stated in [3] and [4]; consequently, the sources of error cancel each other out.

Equation 3 Pseudoranges for Receivers A and B

$$\begin{gathered}P_A^s(t) = \rho_A^s(t) + c * \tau_A(t) - c * \tau^s(t) - d_{iono}^{s} + d_{trop}^{s} + \epsilon_A^s \\ P_B^s(t) = \rho_B^s(t) + c * \tau_B(t) - c * \tau^s(t) - d_{iono}^{s} + d_{trop}^{s} + \epsilon_B^s\end{gathered} \tag{3}$$

According to Rhedgecock [5], the difference between the pseudoranges obtained by two receivers with respect to the same satellite in a single line of sight is equivalent to the distance vector projection between receivers in the satellite direction, as shown in Figure 2.

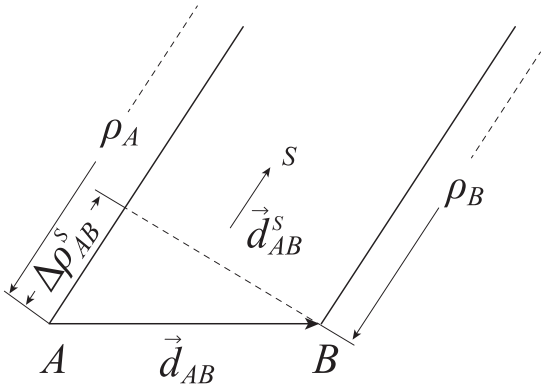


Figure 2. Geometric interpretation of the single-differencing operation.
Source: adapted from [24]

So, as the distance between the receivers and the satellite is much larger than the distance between receivers, the distance vector projection can be considered equal for both receivers. In this way, Rhedgcock's approach is useful for cosines direction of distance between pairs of receivers, within the Weighted Least Squares - Differential Cooperative Positioning (WLS-DCP) approach presented in the next section.

2.6 Cooperative Positioning

Contrastingly, the approach of cooperative positioning is aimed at using data exchanges between GNSS receivers, as an alternative to improve positioning accuracy in each one of the interacting devices.

This approach provides the foundation for the hereby study, whose main contribution is presented in the next section.

2.7 Related Work and Applications

The use of satellite positioning systems for location applications within urban and suburban environments is a market of ongoing growth. This is due to the fact that applications such as public transport, quality and the cost of mobility services are criteria on which costs and demand for public service may be defined [6]. Even innovative applications such as cash on delivery (COD) or the use of unmanned aerial vehicles (UAV), require reliable navigation systems for their operation in this type of environment [7]. With regard to military applications as well as search and rescue operations in situations surrounding natural disasters, they have been well received since the beginning of the 21st century [8], [9], [10]. The Department of Defense of the

United States and the International Space Agency, have seen a growth with the use of these new types of aerospace vehicle since 2005 [11].

A good part of research on this issue has focused on the development of protocols for the exchanging of data between devices with the purpose of making the information of the environment available for any type of application. Since the emergence of sensor networks (Wireless Sensor Networks - WNS), the number of groundwork cases looking into their applicability to satellite navigation systems has increased. The work in [12], discusses the opportunities of the Galileo positioning system for the development of security and authentication solutions backed by GNSS technology, and [13] proposes the characterization of user's location and movement to allow for the recommendation of touristic sites and shopping centers.

[14], notes the development of cooperative positioning systems for Intelligent Transport Systems (ITS), in which relative positioning plays an important role, since the level of positioning accuracy in urban areas is a crucial factor for the navigation of autonomous vehicles in order to avoid car and pedestrian accidents.

In contrast, within the studies reported in literature about positioning techniques based on collaborative work of GPS sensors, is the case of [15] that reports simulation results of three collaborative navigation techniques in urban environments, highlighting the use of a technique called "Relative Vector", in which a known location and coordinate device is used by a second device as a reference point to determine its position, mitigating the impact of low visibility of satellites and multipath effect.

Regarding the works focused on the development of positioning techniques and positioning algorithms for the improvement of accuracy, the case study in Salós [16] can be mentioned. It focused on the simulation of an electronic toll system supported by GNSS systems. In this publication, we use the numerical method of Weighted Least Squares (WLS) along with Receiver Autonomous Integrity Monitoring (RAIM) in order to speed up toll payment and to make the infrastructure necessary for this purpose more flexible.

Of the works found in this field, Du's research is highlighted, [17], in which an outdoor positioning mechanism based on WiFi technology is proposed to complement the accuracy of satellite positioning systems. The focus and results of Du's research are considered a good option to complement the development presented in this publication.

[18] mentions alternatives for the exchange of positioning information between devices from a cooperative point of view. Mahmound suggests the use of a communication infrastructure for short distance measurements (Dedicated Short Range Communications – DSRC; which would work to complement information on

the dynamics of digital receivers and maps. Under these conditions, Mahmound's work simulations conclude that it is possible to reach error levels of 1 - 2 meters for positioning devices inside of urban environments.

From the point of view of unmanned aereal vehicles (UAVs), Fu's work, cited in [19], proposes two strategies of cooperative positioning for a group of UAVs. The first one is focused on the work of a leading device, which is responsible for receiving and merging the information from each device around it so that they can later use the position of the leader as a high precision reference in order to perform their flight maneuvers. This type of strategy is known as centralized cooperative positioning, which has been proposed as a communication strategy for operation of rescue robots in [20], cooperative flying of UAVs [21] and positioning in sensor communication networks WSN [22].

In the second strategy suggested by Fu, each device calculates its position and the error derived from this estimate so that the leader can use this information to estimate the position of all the members of the group with greater precision. This technique, also called Distributed Cooperative Positioning, has been addressed by [23] for the location of UAVs with reference to the ground-based leading robot. Likewise, the distributed strategy is used in [24], for the navigation of a group of robots and their leader in an indoor environment using WiFi technology.

In this paper, an approach to a cooperative positioning algorithm is described, where a set of devices exchange GPS satellite visibility and distance estimations with nearby devices in order to increase their positioning accuracy. Some ionospheric models can be used by single frequency receivers to predict VTEC in their own locations. Section II introduces some theorical background about positioning techniques and the pseudorange model. Section III describes the basis and considerations of the proposed algorithm along with the experimental setup and metrics defined for measuring acurracy improvement levels. Section IV presents and discusses the results and section V summarizes the conclusions.

3. Materials and Methods

It is known that the precision level for positioning tasks is acceptable for ideal open sky conditions. However, not all environments offer this possibility to receivers since such urban environments or areas with high electromagnetic interference are conditions in which the positioning accuracy of GNSS devices is strongly affected.

In this scenario, cooperative positioning emerges as an alternative for GNSS positioning tasks. This paradigm allows GNSS receivers to exchange information, facilitating the improvement of positioning accuracy levels.

Our approach of an algorithm for the cooperative positioning between the GNSS receivers, known as WLS-DCP, is the main contribution in this publication. WLS-DCP relies on the numerical technique of weighted least-square WLS, to take into consideration the elevation of the satellites with the purpose of giving priority to satellite information with less interference and ionospheric delay.

3.1 Least Squares Method

The Least Squares (LS) approach for GPS positioning establishes a linearization process to the nonlinear model of the existing pseudorange between a GPS receiver and a satellite visible from its location, whereby a GPS observation obtained by the receiver can be represented as:

Equation 4 Initial Approach for Least Squares Deduction

$$\begin{aligned} P_{obs} &= \rho_{model} + noise \\ P_{obs} &= P(x,y,z,\tau)|_{(x_0,y_0,z_0,\tau_0)} + \eta \end{aligned} \tag{4}$$

Applying the Taylor series expansion to the pseudo-range model, around an operation point (x_0, y_0, z_0, τ_0), and ignoring higher-order terms and truncating associated errors, the pseudo-range model is defined according to expression (5)

Equation 5 Taylor expansion for pseudorange model

$$P(x,y,z,\tau) = P(x_0,y_0,z_0,\tau_0) + (x-x_0)\frac{\partial P}{\partial x} + (y-y_0)\frac{\partial P}{\partial y} + (z-z_0)\frac{\partial P}{\partial z} + (\tau-\tau_0)\frac{\partial P}{\partial \tau} \tag{5}$$

$$P(x,y,z,\tau) = P(x_0,y_0,z_0,\tau_0) + \frac{\partial P}{\partial x}\Delta_x + \frac{\partial P}{\partial y}\Delta_y + \frac{\partial P}{\partial z}\Delta_z + \frac{\partial P}{\partial \tau}\Delta_\tau$$

By means of establishing the difference (error) between observable values of distance $P(x, y, z, \tau)$ and the pseudo-range linearized model $P(x_0, y_0, z_0, \tau_0)$, we get the next expression:

Equation 6 Matricial version of Least Squares

$$\Delta P = \left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z}, \frac{\partial P}{\partial \tau}\right)(\Delta_x, \Delta_y, \Delta_y, \Delta_\tau)^T\Big|_{(x_0,y_0,z_0,\tau_0)} = H * \Delta X \tag{6}$$

Where:

- $\Delta P = P(x, y, z, \tau) - P(x_0, y_0, z_0, \tau_0)$
- H is the Matrix of Direction Cosines
- $\Delta X = (\Delta_x, \Delta_y, \Delta_y, \Delta_\tau)^T$

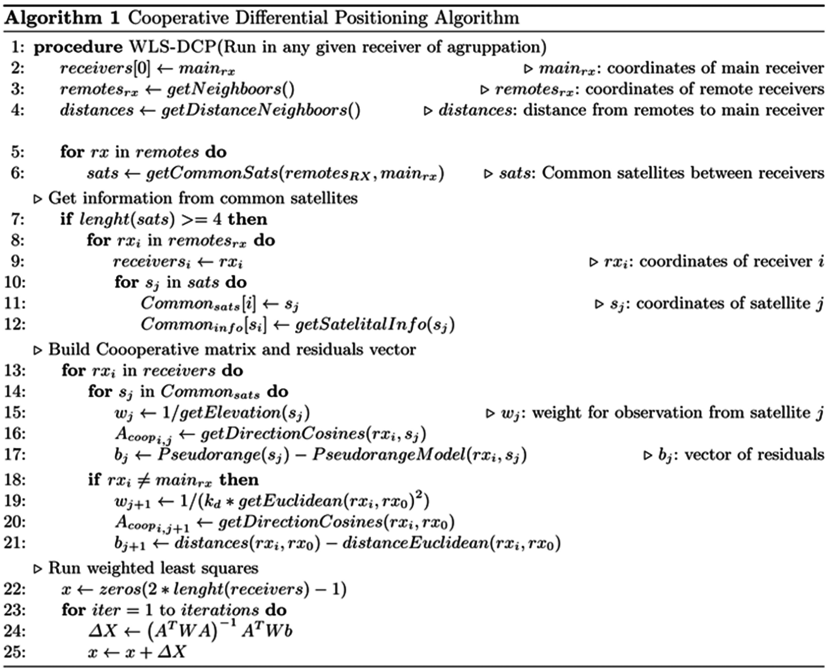
Therefore, the LS method iteratively calculates the value of the term ΔP until the minimum difference is obtained in ΔX that satisfies the expression (7).

Equation 7 Error expression for Least Squares

$$\varepsilon = \min\left(\Delta P - H * \Delta X\right) \tag{7}$$

3.2 Proposed Algorithm for the Approach to Cooperative LS

The approach for the WLS-DCP algorithm starts by using the distance between two GPS receivers to increase the accuracy in the positioning of the same receivers. Additionally, this distance can be considered as additional data, which can be obtained with the support of the remote sensors in the GPS receivers.



```
Algorithm 1 Cooperative Differential Positioning Algorithm
procedure WLS-DCP(Run in any given receiver of agruppation)
    $receivers[0] \leftarrow main_{rx}$    ▷ $main_{rx}$: coordinates of main receiver
    $remotes_{rx} \leftarrow getNeighboors()$    ▷ $remotes_{rx}$: coordinates of remote receivers
    $distances \leftarrow getDistanceNeighboors()$    ▷ $distances$: distance from remotes to main receiver

    for $rx$ in $remotes$ do
        $sats \leftarrow getCommonSats(remotes_{RX}, main_{rx})$    ▷ $sats$: Common satellites between receivers
  ▷ Get information from common satellites
    if $lenght(sats) >= 4$ then
        for $rx_i$ in $remotes_{rx}$ do
            $receivers_i \leftarrow rx_i$    ▷ $rx_i$: coordinates of receiver $i$
            for $s_j$ in $sats$ do
                $Common_{sats}[i] \leftarrow s_j$    ▷ $s_j$: coordinates of satellite $j$
                $Common_{info}[s_i] \leftarrow getSatelitalInfo(s_j)$
  ▷ Build Coooperative matrix and residuals vector
    for $rx_i$ in $receivers$ do
        for $s_j$ in $Common_{sats}$ do
            $w_j \leftarrow 1/getElevation(s_j)$    ▷ $w_j$: weight for observation from satellite $j$
            $A_{coop_{i,j}} \leftarrow getDirectionCosines(rx_i, s_j)$
            $b_j \leftarrow Pseudorange(s_j) - PseudorangeModel(rx_i, s_j)$    ▷ $b_j$: vector of residuals
        if $rx_i \neq main_{rx}$ then
            $w_{j+1} \leftarrow 1/(k_d * getEuclidean(rx_i, rx_0)^2)$
            $A_{coop_{i,j+1}} \leftarrow getDirectionCosines(rx_i, rx_0)$
            $b_{j+1} \leftarrow distances(rx_i, rx_0) - distanceEuclidean(rx_i, rx_0)$
  ▷ Run weighted least squares
    $x \leftarrow zeros(2 * lenght(receivers) - 1)$
    for $iter = 1$ to $iterations$ do
        $\Delta X \leftarrow \left(A^T W A\right)^{-1} A^T W b$
        $x \leftarrow x + \Delta X$
```

Figure 4. Pseudocode of WLS-DCP Algorithm.
Source: own work

Our approach considers that a cluster in cooperative mode is composed of a master receiver and a set of remote receivers. The master receiver is taken as a reference point against which the distance is measured or obtained from each one of the remote receivers that are part of the cluster.

Similarly, any receiver in the cluster can be used as the reference point by the other receivers to improve their accuracy level on their own.

Additionally, the following considerations are taken into account:

- The distance between a couple of receivers d_{AB} ≤ 10km.
- The number of $n_{sat} \geq 4$.

Where:

- n_{sat} is the number of satellites visible in common for the set of receivers.
- d_{AB} is the distance between receivers A and B respectively.

3.3 Assumed Pseudorange Model

The pseudo-range model ρ_j in this study considers the distance geometry to the visible satellite, the clock bias at receiver and the error term η_j^i, where errors associated with ionospheric and tropospheric phenomena and others are clustered.

For demonstration purposes we assume that $\eta_j^i = 0$ for all receivers that make up the cooperative positioning group. Specifically, the pseudoranges to four satellites from a receiver A will be:

Equation 8 Pseudoranges from receiver A to four satellites

$$\begin{bmatrix} \widehat{\rho_A^1} \\ \widehat{\rho_A^2} \\ \widehat{\rho_A^3} \\ \widehat{\rho_A^4} \end{bmatrix} = \begin{bmatrix} \sqrt[2]{(x^1 - x_A)^2 + \left(y^1 - y_A\right)^2 + (z^1 - z_A)^2} + c * (\tau^1 - \tau_A)^2 + \eta_A^1 \\ \sqrt[2]{(x^1 - x_A)^2 + \left(y^1 - y_A\right)^2 + (z^1 - z_A)^2} + c * (\tau^1 - \tau_A)^2 + \eta_A^1 \\ \sqrt[2]{(x^1 - x_A)^2 + \left(y^1 - y_A\right)^2 + (z^1 - z_A)^2} + c * (\tau^1 - \tau_A)^2 + \eta_A^1 \\ \sqrt[2]{(x^1 - x_A)^2 + \left(y^1 - y_A\right)^2 + (z^1 - z_A)^2} + c * (\tau^1 - \tau_A)^2 + \eta_A^1 \end{bmatrix} \tag{8}$$

3.4 Matrix of Direction Cosines

The direction cosines matrix is derived from the receiver's coordinates and the coordinates of each visible satellite. For demonstration purposes, it is considered that a receiver A has visibility to 4 satellites in common, located in positions $(x_i, y_i, z_i, \tau_i) \therefore$ $x = 1 \ldots 4$ as shown in

Equation 9 Matriz of directions cosines for receiver A

$$H_A = \begin{bmatrix} \widehat{H_{x_A}^{0}} & \widehat{H_{y_A}^{0}} & \widehat{H_{z_A}^{0}} & 1 \\ \widehat{H_{x_A}^{1}} & \widehat{H_{y_A}^{1}} & \widehat{H_{z_A}^{1}} & 1 \\ \widehat{H_{x_A}^{2}} & \widehat{H_{y_A}^{2}} & \widehat{H_{z_A}^{2}} & 1 \\ \widehat{H_{x_A}^{3}} & \widehat{H_{y_A}^{3}} & \widehat{H_{z_A}^{3}} & 1 \end{bmatrix} \qquad (9)$$

Up to this point, the direction cosines matrix approach and pseudorange model is similar to what can be considered the solution in autonomous positioning for each clustered receiver by means of LS.

3.5 The Distance Data

For purposes of the mathematical approach, the error on this estimate of distance is considered negligible $\in = 0$. Therefore, the estimated distance between two receivers to $A\ (x_A, y_A, z_A, \tau_A)$ and $B\ (x_B, y_B, z_B, \tau_B)$ will be defined as:

Equation 10 Euclidean distance between receiver A and B

$$d_{AB} = \sqrt[2]{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2} \qquad (10)$$

Expression (9) will be part of the residual vector used for cooperative positioning.

3.6 Direction Cosine associated with the distance between Pairs of Receivers

Now, the approach for the proposed cooperative positioning algorithm takes into account the approach of [5] and the distance data identified in equation (10), modifying the structure of the matrix of direction cosines in order to include the direction cosines associated with the distance from the main receiver to the remote receivers.

For demonstration purposes, it is assumed that the cooperative duo is formed of receptors A and B, where A acts as the main receiver and B is the remote receiver.

Then, the geometric distance formed by A and B is set in equation (10) and the respective direction cosine in $\widehat{d^x_{AB}}$, $\widehat{d^y_{AB}}$, $\widehat{d^z_{AB}}$ are:

Equation 11 Direction cosines for distance between receiver A and B

$$\begin{aligned}
\widehat{d^x_{AB}} &= \frac{(x_B - x_A)}{\sqrt[2]{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}} \\
\widehat{d^y_{AB}} &= \frac{(y_B - y_A)}{\sqrt[2]{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}} \qquad (11) \\
\widehat{d^z_{AB}} &= \frac{(z_B - z_A)}{\sqrt[2]{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}
\end{aligned}$$

Considering the above, the direction cosines matrix in the remote receiver B shall be:

Equation 12 Matrix of direction cosines for receiver B

$$H_B = \begin{bmatrix}
\widehat{H^0_{x_A}} & \widehat{H^0_{y_A}} & \widehat{H^0_{z_A}} & 1 \\
\widehat{H^1_{x_A}} & \widehat{H^1_{y_A}} & \widehat{H^1_{z_A}} & 1 \\
\widehat{H^2_{x_A}} & \widehat{H^2_{y_A}} & \widehat{H^2_{z_A}} & 1 \\
\widehat{H^3_{x_A}} & \widehat{H^3_{y_A}} & \widehat{H^3_{z_A}} & 1 \\
\widehat{d^x_{AB}} & \widehat{d^y_{AB}} & \widehat{d^z_{AB}} & 0
\end{bmatrix} \qquad (12)$$

With these useful considerations, the numerical algorithm is able to minimize the error in coordinates of receivers along with the error in the distance estimated between the paired receivers. This happens in view of the fact that it establishes a useful geometrical relationship that includes the paired receiver's coordinates in such a way that any estimation of the remote receiver coordinates will positively affect the estimation of the main receiver.

3.7 Direction Cosine Matrix for Cooperative Approach

The cooperative matrix of direction cosines is derived from all the matrices of the receiver's direction cosines in the cooperative group. This matrix is formed by locating each of the receiver's matrices diagonally, as can be seen in equation (13).

The number of rows in the cooperative direction cosines matrix is $Rows = n_{sats} * n_{rx} + (n_{rx} - 1)$. Regarding the direction cosines matrices of the remote receivers, they will always have an additional line. The number of columns of the cooperative matrix of cosines is $Cols = n_{vars} * n_{rx}$.

Where n_{sats} is the number of common visible satellites for the group of receivers, n_{rx} is the number of receivers and n_{vars} is the number of variables to solve (x, y, z, τ) in each iteration of WLS-DCP algorithm.

For illustrative purposes, consider the interaction between three receivers (n_{rx} = 3), and four satellites in common (n_{rx} = 4), whereupon the the matrix of cooperative direction cosines would be:

Equation 13 Direction cosine matrix for cooperative approach

$$H_{coop} = \begin{bmatrix} \begin{bmatrix} \widehat{H_{x_A}^{0}} & \widehat{H_{y_A}^{0}} & \widehat{H_{z_A}^{0}} & 1 \\ \widehat{H_{x_A}^{1}} & \widehat{H_{y_A}^{1}} & \widehat{H_{z_A}^{1}} & 1 \\ \widehat{H_{x_A}^{2}} & \widehat{H_{y_A}^{2}} & \widehat{H_{z_A}^{2}} & 1 \\ \widehat{H_{x_A}^{3}} & \widehat{H_{y_A}^{3}} & \widehat{H_{z_A}^{3}} & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \widehat{H_{x_A}^{0}} & \widehat{H_{y_A}^{0}} & \widehat{H_{z_A}^{0}} & 1 \\ \widehat{H_{x_A}^{1}} & \widehat{H_{y_A}^{1}} & \widehat{H_{z_A}^{1}} & 1 \\ \widehat{H_{x_A}^{2}} & \widehat{H_{y_A}^{2}} & \widehat{H_{z_A}^{2}} & 1 \\ \widehat{H_{x_A}^{3}} & \widehat{H_{y_A}^{3}} & \widehat{H_{z_A}^{3}} & 1 \\ \widehat{d_{AB}^{x}} & \widehat{d_{AB}^{y}} & \widehat{d_{AB}^{z}} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \widehat{H_{x_k}^{0}} & \widehat{H_{y_k}^{0}} & \widehat{H_{z_k}^{0}} & 1 \\ \widehat{H_{x_k}^{1}} & \widehat{H_{y_k}^{1}} & \widehat{H_{z_k}^{1}} & 1 \\ \widehat{H_{x_k}^{2}} & \widehat{H_{y_k}^{2}} & \widehat{H_{z_k}^{2}} & 1 \\ \widehat{H_{x_k}^{3}} & \widehat{H_{y_k}^{3}} & \widehat{H_{z_k}^{3}} & 1 \\ \widehat{d_{Ak}^{x}} & \widehat{d_{Ak}^{y}} & \widehat{d_{Ak}^{z}} & 0 \end{bmatrix} \end{bmatrix} \tag{13}$$

3.8 Vector of Residuals

The vector of residuals ΔP appends all differences between psuedoranges measured and its respective pseudoranges modelled for all receiver pairs working in cooperative mode (ex: where A and B are a cooperative duo).

Equation 14 Residuals vector for cooperative approach.

$$\begin{bmatrix} \widehat{\Delta P_A^1} \\ \widehat{\Delta P_A^2} \\ \widehat{\Delta P_A^3} \\ \widehat{\Delta P_A^4} \\ \widehat{\Delta P_B^1} \\ \widehat{\Delta P_B^2} \\ \widehat{\Delta P_B^3} \\ \widehat{\Delta P_B^4} \\ \widehat{\Delta d_{AB}} \end{bmatrix} = \begin{bmatrix} P_A^1 - \sqrt[2]{(x^1-x_A)^2+\left(y^1-y_A\right)^2+(z^1-z_A)^2} + c*(\tau^1-\tau_A)^2+\eta_A^1 \\ P_A^2 - \sqrt[2]{(x^2-x_A)^2+\left(y^2-y_A\right)^2+(z^2-z_A)^2} + c*(\tau^2-\tau_A)^2+\eta_A^2 \\ P_A^3 - \sqrt[2]{(x^3-x_A)^2+\left(y^3-y_A\right)^2+(z^3-z_A)^2} + c*(\tau^3-\tau_A)^2+\eta_A^3 \\ P_A^4 - \sqrt[2]{(x^4-x_A)^2+\left(y^4-y_A\right)^2+(z^4-z_A)^2} + c*(\tau^4-\tau_A)^2+\eta_A^4 \\ P_B^1 - \sqrt[2]{(x^1-x_B)^2+\left(y^1-y_B\right)^2+(z^1-z_B)^2} + c*(\tau^1-\tau_B)^2+\eta_B^1 \\ P_B^2 - \sqrt[2]{(x^2-x_B)^2+\left(y^2-y_B\right)^2+(z^2-z_B)^2} + c*(\tau^2-\tau_B)^2+\eta_B^2 \\ P_B^3 - \sqrt[2]{(x^3-x_B)^2+\left(y^3-y_B\right)^2+(z^3-z_B)^2} + c*(\tau^3-\tau_B)^2+\eta_B^3 \\ P_B^4 - \sqrt[2]{(x^4-x_B)^2+\left(y^4-y_B\right)^2+(z^4-z_B)^2} + c*(\tau^4-\tau_B)^2+\eta_B^4 \\ d_{AB} - \sqrt[2]{(x_B-x_A)^2+\left(y_B-y_A\right)^2+(z_B-z_A)^2} \end{bmatrix} \qquad (15)$$

Additionally, the term associated with the direction cosine of the distance between the receivers is part of the vector of residuals. Therefore, for each pair of receivers that form the cooperative level, the vector of residuals will have an additional row which establishes the difference between the distance and the estimate of their respective algebraic model.

3.9 Weights for Least Squares

WLS is an estimation technique which weighs the observations proportional to the reciprocal of the error variance for that observation and solves the issue of non-constant variance caracteristic in positioning task problems.

So for WLS-CDP positioning algorithm, it is assumed that the satellite with the highest elevation among the set of satellites establishes the weight of all the elements of the matrix. The value placed on the diagonal of the matrix, which corresponds to the direction cosine's distance location, is supported by the exponential law presented by Rhedgecock's proposal. The rest of the elements located in the row and column of this place are equal to zero.

Equation 15 Error of precision from Rhedgecock's approach

$$\varepsilon = 2.476x10^{-8}x^2 \qquad (16)$$

Therefore, it is considered that the weight of the direction cosine of distance is related to the length of the baseline between receivers. This applies to all common visible satellites between each cooperative pair formed between the main receiver and remote receivers.

3.10 Metrics of Improvement

The definition of a metric or indicator supports and allows us to evaluate when WLS-CDP improves positioning accuracy of a GPS receiver with respect to its position in autonomous mode.

The *improvement index* is defined as:

The average of the difference between the positioning error in standalone mode $E(i)_{Stand}$ *and* $E(i)_{Coop}$, *for each of the recipients of a cluster; for a set of* n *satellite observations.*

It is important to mention that:

- A cluster is a set of receivers that collects data from a group of satellites in common to execute the cooperative differential positioning algorithm.
- The cooperative average error $\Delta\widehat{E_{Stand}^{Coop}}$, is obtained from averaging the difference between autonomous positioning errors $E(i)_{Stand}$ and cooperative errors $E(i)_{Coop}$ of a GPS receiver, for a period of time. (eg: n observed in a GPS day).
- The improvement index l_{pos} refers to the extent to which positioning accuracy of a GPS receiver improves, when it exchanges satellite information with its closest neighbors.

3.11 The Average Cooperative Error

The average difference between the positioning error in cooperative and autonomous scope $\Delta\widehat{E_{Stand}^{Coop}}$, is obtained considering that each receiver in the cluster has n samples of GPS observations from common satellites with its neighbors. Thus, the value of $\Delta\widehat{E_{Stand}^{Coop}}$, for each receiver can be obtained through:

Equation 16 The average cooperative error

$$\Delta\widehat{E_{Stand}^{Coop}} = \frac{\sum_{i=1}^{n}(E(i)_{Stand} - E(i)_{Coop})}{n} \qquad (17)$$

In the equation above, $E(i)_{Stand}$ refers to the difference between the real coordinates of the receivers in the coordinates system ECEF[1] and the standalone mode positioning solution obtained by means of the LS algorithm. Likewise, $E(i)_{Coop}$ is obtained in cooperative mode.

3.12 The Improvement Index

The improvement index l_{pos} is obtained from averaging the value $\Delta\widehat{E(J)}_{Stand}^{Coop}$ for the receivers comprising a cluster of size (m). The value of l_{pos} works as an indicator of how much position accuracy of a GPS receiver improves with respect to its position in standalone mode.

The index improvement is defined as:

Equation 17 The improvement index

$$I_{pos} = \frac{\sum_{j-1}^{m}(\Delta\widehat{E(J)}_{Stand}^{Coop} - \overline{\Delta E(J)_{Stand}^{Coop}})}{m} \qquad (18)$$

So, if the value of $l_{pos} > 0$ is positive, it indicates that the standalone positioning error is greater compared with the error retrieved in cooperative mode. Therefore, the scenarios with a positive rate of improvement are considered favorable scenarios for the WLS-DCP algorithm. For cases unfavorable to the hypothesis, the conditions and scenarios under which the index $l_{pos} < 0$ is negative are considered.

4. Methodology

With the purpose of evaluating the proposed cooperative algorithm, a set of GNSS reference stations located in medium and low latitudes have been selected, for prevalidating the hypothesis of improvement in precision positioning on these latitudes.

For this purpose, we have established a group of stations with important features to evaluate the cooperative positioning approach proposed in this publication in a global way.

The stations in Table 1, are categorized by range of separation ranging from 100m to approximately 10km so that the impact of the baseline length can be evaluated

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[1] ECEF: Earth-Centered, Earth-Fixed (ECEF) coordinate system. It is a Cartesian coordinate system whose reference point is the center of the earth. [29]

on the positioning accuracy of the devices, for which the behavior is expected to be similar to the positioning techniques DGPS and RTK.

Additionally, since the stations are at medium and low latitudes (where the impact of the ionosphere is less negligible), the selected stations allow for the identificacion of ionospheric correction conditions and models that contribute to the accuracy for differential cooperative positioning. In the presented scenarios, each receiver uses an ionospheric correction model similar to or different from their neighboring devices, aside from using the satellite information that they provide to facilitate the calculation of their own position.

Table 1. GNSS Stations used in the study.

Group	Receivers	$\text{Size}_{\text{group}}$	Location
1	(ljrn, p553, p554, fzhs)	4	USA
2	(CN20, TGPM)	2	Pánama
3	(TGMX, UNPM)	2	México

Source: own work

On the other hand, research that has given rise to this publication have, as their final objective, to evaluate cooperative positioning using GNSS single-frequency GNSS devices immersed in urban canyons of Colombian cities. Therefore, the results of this publication allow us to estimate the impact of the ionospheric correction models on positioning accuracy for the stations mentioned above, located in mid-low latitudes. This serves as a preliminary result to validate the hypothesis of the research work behind this publication.

4.1 Simulation scenarios

The experimental design includes the use of observables C1 or P1. In addition, the distance between each pair of receivers is considered as an observable that can present noise levels in its associated measurement. Then, a set of three scenarios have been defined concerning the variation of the noise levels in the distance data along with eight possible combinations of the ionospheric models to use on main or remotes receivers. This is done to assess their impact level on positioning accuracy of receivers working in cooperative mode.

Therefore, GPS days 10, 11, 12, 13 of year 2017 have been assessed in order to determine whether ionospheric variation has an impact on the algorithm solution. Then, the scenarios selected for the study are defined in the following section.

4.2 Variation of the ionospheric correction model

The ionospheric models and blending of each one of the scenarios are:

Scenario 1: without ionospheric correction. For this scenario, neither the main nor the remote receivers use any correction model to mitigate the ionospheric effects.

Scenario 2: Differential #1 It emulates the Real Time Kinematic positioning (RTK) behavior, where a set of receivers get the available information from a receiver that has double frequency corrections to improve its positioning.

Scenario 3: Differential #2 Unlike the Differential #1 scenario, its conditions imply that the remote receivers handle the simple Klobuchar frequency correction model.

Scenario 4: Klobuchar For this scenario, the accuracy level is evaluated when all receivers use the simple Klobuchar frequency correction model.

Equation 18 Klobuchar correction model

$$I_{L1_{delay}} = \begin{cases} \left[5 * 10^{-9} + A\left(1 - {x^2}/{2} + {x^4}/{24}\right)\right] * F & \therefore if \ \|\chi\| \le 1.57 \\ 5 * 10^{-9} * F & \therefore \ if \ \|\chi\| > 1.57 \end{cases} \qquad (15)$$

Where:

- $F = 1 + 16 * (0.53 - e)^3$
- $A = \sum_{n=0}^{3} \beta_n \Phi_m^n$, is an expression to get the amplitude of ionosphere delay.
- χ Represents the phase of ionospheric delay.
- α_n, β_n are coefficients broadcasted in the GPS satellite navigation message.
- Φ_m Represents the geomagnetic latitude of the ionosferic piercing point.

The Klobuchar Ionospheric model in equation (15) is adopted by single frequency GPS receivers to correct the ionospheric delay of the L1 carrier. It is defined as a single layer ionospheric model (SLM - Single Layer Model), because the ionosphere (i.e. its TEC) is supposedly concentrated in an infinitesimally thin layer placed at an average altitude of 350 km set by the Earth's surface. More details about Klobuchar model can be found in [25] or [26].

Scenario 5: Differential #3 unlike the Differential # 2 scenario, the conditions of this scenario imply that the remote receivers handle the Klobuchar simple frequency correction model.

Scenario 6: Standard its conditions imply that the receivers are the standard model.

Equation 19 Standard ionospheric correction model

$$M_{STEC} = \frac{STEC}{VTEC} = \frac{1}{\cos(z)} \qquad (16)$$

- **STEC** Slant total electron content when EM ray travels through the ionosphere along the radial direction.
- **VTEC** Vertical total electron content is the special case of STEC where the EM ray travels through the ionosphere along the radial direction.
- $\boldsymbol{M_{STEC}}$ Mapping functions allows estimate STEC from VTEC.
- **z** Piercing angle for a slanted ray travelling through shell of ionosphere model.

Scenario 7: Diferencial #4 unlike the Differential #3 scenario, its conditions imply that the remote receivers handle the thin shell model.

Scenario 8: Thin Shell Ionospheric Model This scheme proposes using the Taylor series expansion for a three-dimensional ionosphere model.

Equation 20 Thin Shell Ionospheric Model

$$M_{STEC} = \frac{STEC}{VTEC} = \frac{1}{\cos(z)} + \frac{\cos^2(z) - 1}{8\,\mathrm{r}_s{}^2 \cos^5(z)} * \mathrm{d}_{iono}{}^2 + \frac{7 - 10\cos^2(z) + 3\cos^4(z)}{128\,\mathrm{r}_s{}^4 \cos^5(z)} * \mathrm{d}_{iono}{}^4 + \cdots \quad (17)$$

- STEC, VTEC, $\boldsymbol{M_{STEC}}$ and **z** are Slant TEC, Vertical TEC, Mapping TEC function and piercing angle respectively.
- $\boldsymbol{r_s}$ Distance to the center of the dense electron layer of the ionosphere above the Earth's surface.
- $\boldsymbol{d_{iono}}$ Thickness of shell ionosphere considered by the model.
- **z** Piercing angle for a slanted ray travelling through shell of ionosphere model.

The study of ionospheric correction and mapping functions has been extensively addressed by Smith et al. in [27]. Smith concludes that achieving high levels of precision requires three-dimensional modelling of the total electron content of the ionosphere. Even so, the mapping function in equation (17) improves the level of correction of the ionospheric delay with respect to the standard shell model by up to 50%.

4.3 Variation in distance data

For all the previous scenarios the variation in noise levels for distance data is performed to evaluate its impact over positioning accuracy of the receivers that works in cooperative mode. The error for distance data ϵ_d takes values of 0%, 5%, 10%

Results

This section shows the analysis and improvement indices obtained for each one of the scenarios where the cooperative differential positioning algorithm was evaluated. The stations sets used to assess each one of the above scenarios are (*cn20, tgmp*), (*ljrn, p553, p554 y fzhs*) and (*unpm, tgmx*) respectively as shown in Table 1.

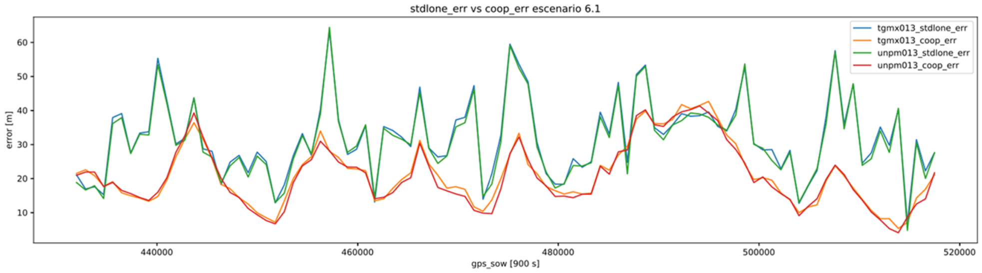
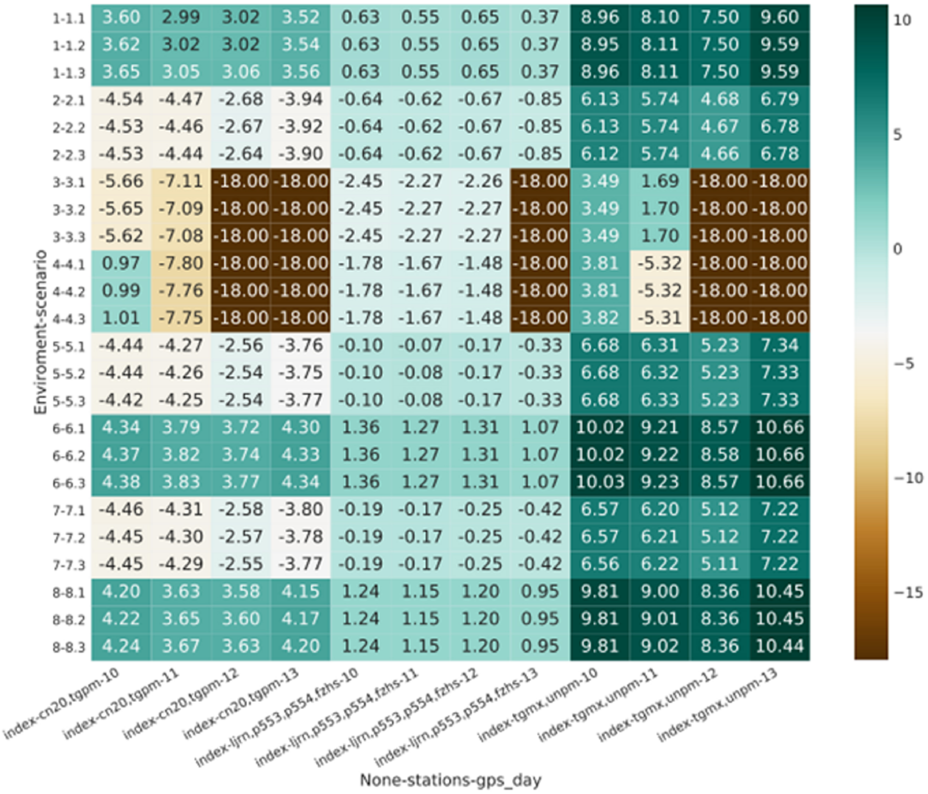


Figure 4. Error in cooperative autonomous positioning for 13 GPS day in (unpm, tgmx).
Source: own work

Figure 5 shows the difference in positioning error levels for the autonomous positioning solution using LS and cooperative WLS-DCP, for the stations that make up the cluster (unpm, tgmx) in the GPS 13 day. It should be noted that the error levels in cooperative mode with the suffixc ($coop\_err$), appear below the error levels obtained by the autonomous solution ($stdlone\_err$) for the positioning solutions obtained throughout the day.

The improvement index shows stable behavior with respect to the variation in the error level in the distance between the main and the other receivers. As can be seen on the heatmap in Figure 5, which shows the obtained results from executing the cooperative positioning algorithm for three cluster of receivers presented in Table 1, taking samples every 4 minutes throughout GPS days 10, 11, 12, 13 in 2017.



Enviroment-scenario	index-cn20,tgpm-10	index-cn20,tgpm-11	index-cn20,tgpm-12	index-cn20,tgpm-13	index-ljrn,p553,p554,fzhs-10	index-ljrn,p553,p554,fzhs-11	index-ljrn,p553,p554,fzhs-12	index-ljrn,p553,p554,fzhs-13	index-tgmx,unpm-10	index-tgmx,unpm-11	index-tgmx,unpm-12	index-tgmx,unpm-13
1-1.1	3.60	2.99	3.02	3.52	0.63	0.55	0.65	0.37	8.96	8.10	7.50	9.60
1-1.2	3.62	3.02	3.02	3.54	0.63	0.55	0.65	0.37	8.95	8.11	7.50	9.59
1-1.3	3.65	3.05	3.06	3.56	0.63	0.55	0.65	0.37	8.96	8.11	7.50	9.59
2-2.1	-4.54	-4.47	-2.68	-3.94	-0.64	-0.62	-0.67	-0.85	6.13	5.74	4.68	6.79
2-2.2	-4.53	-4.46	-2.67	-3.92	-0.64	-0.62	-0.67	-0.85	6.13	5.74	4.67	6.78
2-2.3	-4.53	-4.44	-2.64	-3.90	-0.64	-0.62	-0.67	-0.85	6.12	5.74	4.66	6.78
3-3.1	-5.66	-7.11	-18.00	-18.00	-2.45	-2.27	-2.26	-18.00	3.49	1.69	-18.00	-18.00
3-3.2	-5.65	-7.09	-18.00	-18.00	-2.45	-2.27	-2.27	-18.00	3.49	1.70	-18.00	-18.00
3-3.3	-5.62	-7.08	-18.00	-18.00	-2.45	-2.27	-2.27	-18.00	3.49	1.70	-18.00	-18.00
4-4.1	0.97	-7.80	-18.00	-18.00	-1.78	-1.67	-1.48	-18.00	3.81	-5.32	-18.00	-18.00
4-4.2	0.99	-7.76	-18.00	-18.00	-1.78	-1.67	-1.48	-18.00	3.81	-5.32	-18.00	-18.00
4-4.3	1.01	-7.75	-18.00	-18.00	-1.78	-1.67	-1.48	-18.00	3.82	-5.31	-18.00	-18.00
5-5.1	-4.44	-4.27	-2.56	-3.76	-0.10	-0.07	-0.17	-0.33	6.68	6.31	5.23	7.34
5-5.2	-4.44	-4.26	-2.54	-3.75	-0.10	-0.08	-0.17	-0.33	6.68	6.32	5.23	7.33
5-5.3	-4.42	-4.25	-2.54	-3.77	-0.10	-0.08	-0.17	-0.33	6.68	6.33	5.23	7.33
6-6.1	4.34	3.79	3.72	4.30	1.36	1.27	1.31	1.07	10.02	9.21	8.57	10.66
6-6.2	4.37	3.82	3.74	4.33	1.36	1.27	1.31	1.07	10.02	9.22	8.58	10.66
6-6.3	4.38	3.83	3.77	4.34	1.36	1.27	1.31	1.07	10.03	9.23	8.57	10.66
7-7.1	-4.46	-4.31	-2.58	-3.80	-0.19	-0.17	-0.25	-0.42	6.57	6.20	5.12	7.22
7-7.2	-4.45	-4.30	-2.57	-3.78	-0.19	-0.17	-0.25	-0.42	6.57	6.21	5.12	7.22
7-7.3	-4.45	-4.29	-2.55	-3.77	-0.19	-0.17	-0.25	-0.42	6.56	6.22	5.11	7.22
8-8.1	4.20	3.63	3.58	4.15	1.24	1.15	1.20	0.95	9.81	9.00	8.36	10.45
8-8.2	4.22	3.65	3.60	4.17	1.24	1.15	1.20	0.95	9.81	9.01	8.36	10.45
8-8.3	4.24	3.67	3.63	4.20	1.24	1.15	1.20	0.95	9.81	9.02	8.36	10.44

Figure 5. Heatmap of improvement index with variation in the distance data.
Source: own work

The variation in I_{pos} for any of these scenarios is minimal or negligible. Yet, this may be due to the fact that the clusters average distance (ljrn, p553, p554 y fzhs) is around 2.5 km. For the pairs (cn20, tgmp) and (unpm, tgmx), it does not exceed 200 m.

According to the previous explanation, it is concluded that the error level in the measured observable of distance between pairs of receivers has no significant influence on the index improvement when separation between the devices is very large, as can be seen in heatmap of Figure 5.

Bearing in mind that stations employed for this study are on the Caribbean, where the margin of error associated with ionospheric delay is significant, evaluation of the influence the ionospheric model has over the receivers' positioning accuracy level using the suggested differential cooperative approach described in this paper has been considered.

The improvement indices for scenarios 6 and 8 are similar and related since Smith's approach in [27], which is a 3D approximation of a single layer ionosphere model, where the standard model could be derived from Smith's model considering that the thickness of the ionosphere is zero.

6. Discussion and conclusions

The results obtained agree with the Hedgecock approach in [24]. Hedgecock says thar the level of error is inversely proportional to the distance that separates the receivers and establishes a technique of differential positioning; our contribution is the extension of differential positioning towards the cooperative field. Additionally, the analysis and results obtained on the ionospheric correction and the precision in cooperative positioning conlude that when the correction model used by GNSS receivers is similar, the accuracy of positioning is high compared with the ionospheric correction models used by different receivers.

Regarding the improvement index obtained, it is in a range from 5m to 10.5m without considering the outliers of the scenarios in which it is not possible to obtain positioning solution in cooperative mode. The index of improvement and the levels of error in positioning agree with the levels reported in [28]. However, a differentiating factor of our work with respect to Susan et al. is that their results employ postprocessing and differential corrections and our approach is aimed at processing near real time.

A factor of similarity is that differential corrections are carried out iteratively with respect to the main receptor of each group of receivers, in a similar way to [28] where differential corrections are calculated with respect to a reference station. Additionally, the stations used by Susan et al. are located in mid-low latitudes, like those used in our study.

A degree of correlation between the ionospheric correction models is observed between the results obtained and presented in Figure 5, from which it follows that using similar ionospheric correction models in all the receivers contributes to the precision of positioning in cooperative mode.

The presented approach contributes to the conception of a cooperative positioning system of low cost, useful for the technification and improvement of the public transport service in Colombian cities, public transport applications, under similar approaches to those of [14].

Based on Figure 5 which summarizes the results obtained for the improvement index in the formulated scenarios for this study, it can be concluded that:

The double frequency ionospheric correction in the differential cooperative positioning scenario is unfavorable, since only the set (*tgmx, unpm*) will improve its precision level using this type of configuration.

The combination of double frequency ionospheric correction and Klobuchar ionospheric model turns out to be the most unfavorable scenario for differential cooperative positioning. In scenarios 3 and 4, around 95% of the improvement index results were negative values, including outliers due to problems of convergence in the algorithm because of the high variance in the residual vector, which could be associated with the difference that exists between the two models of ionospheric correction.

In scenarios 5 and 7, the double frequency correction on the main receiver is proposed. There, two of the three clusters fail to improve positioning accuracy. Only the cluster (*ljrn, p553, p554, fzhs*) has a disadvantage because it has the most distant receivers. This implies a higher level of error in the estimate of the baseline length obtained by the DCP-WLs algorithm after each iteration. Only the couple (*tgmx, unpm*) presents positive improvement indices (blue color), for the majority of the evaluated scenarios. This could be associated with two factors: the first one is the distance between the receivers of this pair, which does not exceed 150m. The second important factor is the location and visibility of satellites in common in this cluster, which implies favorable conditions for the positioning task.

The most favorable scenarios for differential cooperative positioning are scenarios 1, 6 and 8. These scenarios have a shared characteristic: the ionospheric correction models used in the main and remotes receivers are the same. Thus, the improvement index for these scenarios is closely related to the distance estimate and not to the influence of ionospheric delay correction.

It is important to mention that the level of noise or uncertanty in the distance data do not show influence when the distance between receiver pairs is large. However, for shorter distances a variable behavior in the improvement index is observed.

Additionally, the formulation and approach of the matrix of director cosines makes it possible to consider measurements from multiple sensors. These measurements are useful for applications that involve unmanned aerial vehicles, which facilitate the transportation of supplies for rescue or military troops in areas of difficult access.

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8. References

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